The Incredible Doubling Lily Pad

Bob & George • Bob Albrecht & George Firedrake • bob@geekclan.com

🏳🏳🏳	Thank you, teachers, tutors, librarians and others who help learners learn math and science. This eBook is especially for you in the hope that you might find it useful and fun for you and your students. This eBook is a fantasy. Reality expands to fill the available fantasies.	🏳🏳🏳

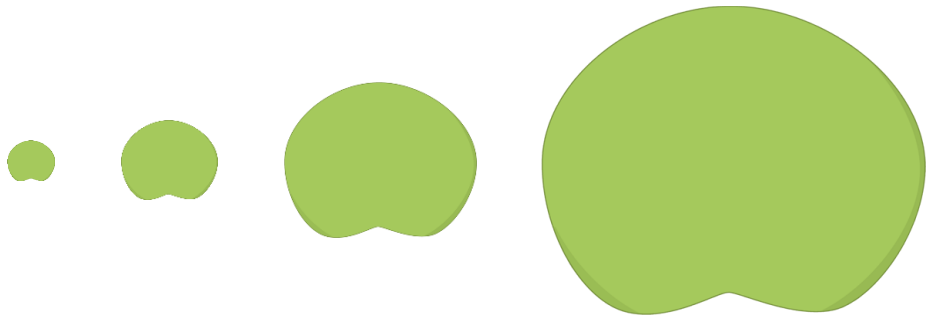
This eBook is a fantasy about an old math problem involving a lily pad floating in a pond. The lily pad doubles in size each month or day or second or other time period and eventually covers the pond. Here there be Internet sites that present variations of the problem:

The expanding lily pad: a retirement problem
http://micawberprinciple.com/the-expanding-lily-pad-a-retirement-riddle-1212/

A lily pad doubles in size every second
https://math.stackexchange.com/questions/2429872/a-lily-pad-doubles-in-area-every-second-after-one-minute-it-fills-the-pond-ho

Question about logic riddle: (patch of lily pads that doubles each day)
https://math.stackexchange.com/questions/1892365/question-about-logic-riddle-patch-of-lily-pads-that-doubles-every-day

There are many more sites. Search using keywords such as lily pad pond double size.



et cetera

The lily pad image is from http://cliparts.co/clipart/2589999, a great source of clip art.
Many lily pad images reside at http://cliparts.co/lily-pad-picture.
Visit Clipart's home page – a cornucopia of clip art! http://cliparts.co/

Double in size? What might that mean? At least three possibilities swirl about in our minds:

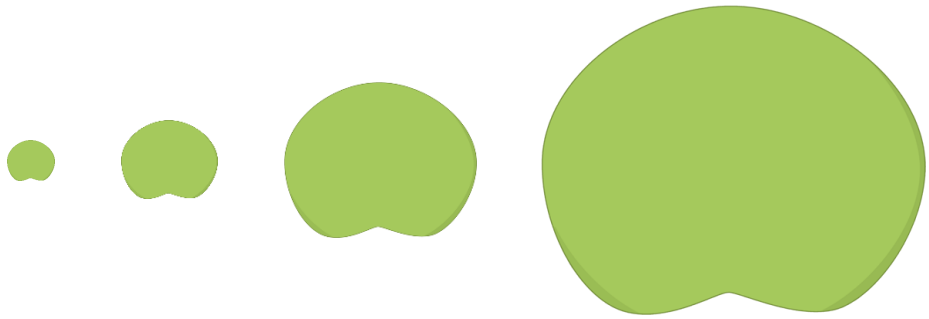
- The lily pad's volume doubles in size every time period.
- The lily pad's area doubles in size every time period.
- The lily pad's linear dimensions such as length, width and height double in size every time period.

We searched the Internet for images of lily pads. They are roundish – approximately circular.

▻▻▻	This eBook is a fantasy, so we will fantasize: We assume (fantasize) that the lily pad is circular and its diameter doubles every time period. This choice leads to fantastic results. Yeah! Huzzah! Notation: *Diameter* or *diameter* (In italics) is a variable. It is the measure of the length of the lily pad's diameter.	▻▻▻

We have seen variations in the time it takes the doubling lily pad to cover the pond:

- Double in size every day; cover the pond in 30 days.
- Double in size every day; cover the pond in 48 days.
- Double in size every second; cover the pond in 1 minute (60 seconds).



and so on

The lily pad image is from http://cliparts.co/clipart/2589999, a great source of clip art.
Many lily pad images reside at http://cliparts.co/lily-pad-picture.
Visit Clipart's home page – a cornucopia of clip art! http://cliparts.co/

We wonder:

- Are lily pads that double in size every day real-life lily pads? We are skeptical.
- Are lily pads that double in size every second magical lily pads? Hogwarts lily pads?
- How big and what shape is the pond? We don't know.

Fantasy # 1

Assumption: The lily pad is approximately circular. Its diameter doubles in length every day.

Assumption: The length of the lily pad's diameter on *day* = 0 is 1 meter. Table 01 shows the length of the diameter of a doubling lily pad that covers the pond in 30 days.

Table 01 Fantasy: length of the diameter of a circular doubling lily pad

Day	***Diameter***
0	1 meter
1	2 meters
2	4 meters
3	8 meters
4	16 meters
5	32 meters
10	1,024 meters
20	1,048,576 meters
28	268,435,456 meters (the pond is 1/4 covered)
29	536,870,912 meters (the pond is 1/2 covered)
30	1,073,741,824 meters (the pond is covered)

Length of the lily pad's diameter at the end of *day* = 30:

- 1,073,741,824 meters
- 1,073,742 kilometers (rounded to one's place)

Wow! That is a big lily pad. And a big pond.

Earth's equatorial circumference is 40,075 kilometers. On *day* = 30, the lily pad's diameter is

- 1,073,742 km / 40,075 km = 26.79 times Earth's equatorial circumference.

Around the world 26.79 times in 30 days! Jules Verne and Phileas Fogg would be incredibly envious.

- Around the world in eighty days
  https://en.wikipedia.org/wiki/Around_the_World_in_Eighty_Days

The mean distance of the Moon from Earth is 384,400 kilometers (384,400,000 meters). On *day* = 30, the lily pad's diameter is

- 1,073,742 km / 384,400 km = 2.79 times mean distance Earth to Moon.

Mythology Note Moon goddess Selene (Greek name), known in Rome as Luna, is surprised to see a lily pad growing towards her domain. How might she react?

- Selene https://en.wikipedia.org/wiki/Selene
- Luna https://en.wikipedia.org/wiki/Luna_%28goddess%29

Math Note – OK to skip. Assume: one end of the lily pad's diameter is on Earth at a location you choose such as your school or your home town or your back yard. On what day does the other end (the far-out end) of the lily pad's diameter cross the Moon's orbit?

During that day, the length of the lily pad's diameter becomes equal to 384,400 kilometers.

- Day 28: Far-out end of the diameter is 268,435 kilometers from Earth, not yet at Moon's orbit.

- Day 29: Far-out end of the diameter is 536,871 kilometers from Earth, beyond Moon's orbit.

Oops! To prevent a Moon-lily pad collision, assume that the lily pad is not in the plane of Moon's orbit so that Moon does not collide with the lily pad as she goes 'round and 'round Earth.

At the end of day 29, the far-out end of the lily pad's diameter is greater than 384,400 km (kilometers) from Earth. To calculate when that happens during day 29, all we or you need do is solve the equation:

- $1 \times 2^{day} = 384{,}400$

Let's do it.

Logarithmic alakazam	**Math property used in logarithmic alakazam**
$\log (1 \times 2^{day}) = \log (384{,}400$	If $A = B$, then $\log (A) = \log (B)$ [$A > 0$, $B > 0$]
$\log (1) + \log (2^{day}) = \log (384{,}400)$	$\log (A \times B) = \log (A) + \log (B)$
$\log (1) + day \times \log 2 = \log (384{,}400)$	$\log (2^{power}) = power \times \log (2)$
$day \times \log 2 = \log (384{,}400)$	$\log (1) = 0$
$day = [\log (384{,}400)] / \log (2) = 28.52$	Division property of equality.
Check: $1 \times 2^{28.52} = 384{,}924$	384,924 km > 384,400 km Yeah!

The far-out end of the lily pad's diameter reaches and passes the distance of Moon's orbit a little past noon on *day* 28. [Noon on day 28 occurs half way through the day at *day* = 28.50.]

Fantasy # 2

Assumption: The lily pad is approximately circular. Its diameter doubles every second.

Assumption: The length of the lily pad's diameter at *time* = 0 second is 1 meter. Table 02 shows the length of the diameter of a doubling lily pad that covers the pond in 60 seconds.

Table 02 Length of the diameter of a doubling lily pad

***Time* (Second)**	***Diameter***
0	1 meter
1	2 meters
2	4 meters
3	8 meters
4	16 meters
5	32 meters
10	1,024 meters
20	1,048,576 meters
30	1,073,741,824 meters
40	1,099,511,627,776 meters
50	1,125,899,906,842,624 meters
58	288,230,376,151,711,744 meters [pond is 1/4 covered]
59	576,460,752,303,423,488 meters [pond is 1/2 covered]
60	1,152,921,504,606,846,976 meters [pond is covered]

Length of lily pad's diameter at *time* = 60 seconds:

- 1,152,921,504,606,846,976 meters
- 1,152,921,504,606,847 kilometers (rounded to one's place)

Incredible! Way beyond wow! That is an astronomical-size lily pad.

The distance of Earth from the Sun is 1 astronomical unit (AU). 1 AU = 149,600,000 km (kilometers).

- Astronomical unit https://en.wikipedia.org/wiki/Astronomical_unit

In astronomical units (AU), the length of the lily pad's diameter at *time* = 60 s (second) is

- 1,152,921,504,606,847 km / 149,600,000 km/AU = 7,706,695 AU

Incredible! If one end of the lily pad's diameter is on Earth, the other end is more than 7.7 million AU from Earth, far beyond the orbit of Pluto, the Kuiper belt and the Oort cloud.

- Kuiper belt https://en.wikipedia.org/wiki/Kuiper_belt
- Oort cloud https://en.wikipedia.org/wiki/Oort_cloud

What say?! Might the light year (ly) be a more appropriate measure of the length of the lily pad's diameter at *time* = 60 seconds? We think so.

- 1 light year (ly) = 9.4607E15 m (meter) Notation: E15 = $\times 10^{15}$
- 1 light year (ly) = 9.4607E12 km (kilometer) Notation: E12 = $\times 10^{12}$

Above we show 1 ly (light year) in scientific notation used in the TI-84 Graphing Calculator (E15 = $\times 10^{15}$, E12 = $\times 10^{12}$) with 5 significant digits, also called significant figures.

- Significant figures https://en.wikipedia.org/wiki/Significant_figures

Below we show the length of the lily pad's diameter at *time* = 60 seconds in TI-84-style scientific notation, rounded arbitrarily and capriciously to 7 significant digits.

- *diameter* = 1.152922E18 m (meter) [7 significant digits]
- *diameter* = 1.152922E15 km (kilometer) [7 significant digits]

What is the length of the diameter of that astronomical-size lily pad in light years (ly)?

- 1.152922E18 m / 9.4607E15 m/ly = 121.86 ly [5 significant digits]
- 1.152922E15 km / 9.4607E12 km/ly = 121.86 ly [5 significant digits]

Amazing! Astronomically awesome! If one end of the lily pad's diameter is on Earth, the other end is approximately 122 ly (light years) from Earth. Far out!

What stars are closer than 122 ly (light years) from Earth, or approximately 122 ly (light years) from Earth? Here is a short list. YOUR STUDENTS can investigate the stars we suggest and find MANY MORE!

Alpha Centauri & Proxima Centauri 4.4 ly [closest stars to the Solar System]
Alpha Centauri https://en.wikipedia.org/wiki/Alpha_Centauri

Sirius 8.6 ly https://en.wikipedia.org/wiki/Sirius
Sirius, known as the 'dog star', is the brightest star in Earth's night sky. [Planets can be brighter.]
Sirius is in the constellation Canis Major https://en.wikipedia.org/wiki/Canis_Major .

Castor 51 ly https://en.wikipedia.org/wiki/Castor_(star)
Pollux 34 ly https://en.wikipedia.org/wiki/Pollux_(star)
The Gemini twins. Greek mythology: sons of Leda. Leda is the mother of Helen of Troy.

Altair 17 ly https://en.wikipedia.org/wiki/Altair
Vega 25 ly https://en.wikipedia.org/wiki/Vega
Chinese/Japanese mythology: weaver princess (Vega) and cowherd (Altair).
See Tanabata festival https://en.wikipedia.org/wiki/Tanabata

Stars of the Big Dipper
https://en.wikipedia.org/wiki/Big_Dipper
Dubhe 124 ly
Merak 79 ly
Phecda 84 ly
Megrez 58 ly
Alioth 81 ly
Mizar 78 ly
Alkaid 101 ly

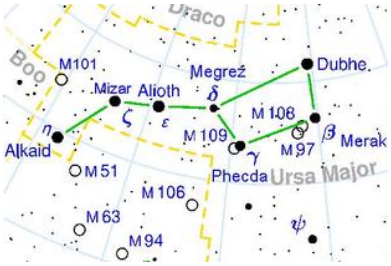


Image of the Big Dipper is from Wikipedia.

The Big Dipper is an asterism in the constellation Ursa Major*.

Close to Mizar is Alcor. Mizar and Alcor are known as the Horse and Rider.

Dubhe and Merak are pointer stars. A line through Merak and Dubhe passes close to Polaris, the North Star.

* Asterism https://en.wikipedia.org/wiki/Asterism_%28astronomy%29
Ursa Major https://en.wikipedia.org/wiki/Ursa_Major

Explore, investigate! Learn about the stars we listed above. Learn about astronomy, Greek mythology, Chinese mythology and Japanese mythology. You and your students can intertwingle math, science, and other areas of knowledge.

YOUR STUDENTS: Renaissance kids? Polymath kids? Intertwingled math, science and other kids?

Oops! The far end of the lily pad's diameter travels faster than light.

Assume: One end of the diameter of a lily pad that doubles in size (diameter) every second is on Earth.

- At *time* = 59 seconds. the far-out end is 576,460,752,303,423,488 m (meters) from Earth.
- At *time* = 60 seconds, the far-out end is 1,152,921,504,606,846,976 m (meters) from Earth.

The far-end of the diameter traveled

- 1,152,921,504,606,846,976 m – 576,460,752,303,423,488 m
  = 576,460,752,303,423,488 m

in 1 second!

Average speed of the far end of the lily pad = 576,460,752,303,423,488 m/s (meters/second)

Oops, that is far faster than the speed of light.

- Speed of light = 299,792,458 m/s (meter/second) [exact]
- Speed of light = 2.99792458E8 m/s (meters/second) [exact]

How many times the speed of light did the lily pad's far end travel during the 60th second?

- 576,460,752,303,423,488 m/s / 299,792,528 m/s = 1,922,865,644 (rounded)

Almost 2 billion times the speed of light! James Kirk and Jean-Luc Picard would love to captain a starship that could travel 2 billion times the speed of light.

Science. Einstein's Theory of Relativity states that nothing can travel faster than the speed of light. The speed of light is the speed limit of the universe. This presents a big problem to authors of books, movies, television and other media.

Fantasy. Ignore science and imagine that objects such as starships and the far end of a lily pad can travel faster that light. Example: The Starship Enterprise's warp speed:

- Warp speed https://sto.gamepedia.com/Warp_speed
- Warp drive https://en.wikipedia.org/wiki/Warp_drive#Warp_velocities
- Warp factor http://memory-alpha.wikia.com/wiki/Warp_factor

Readers and watchers of books, movies, television and other media:

- If you are science-ignorant, enjoy.
- If you are science-savvy, suspend your belief system and enjoy.

We suggest explorations/investigations for your students

Browse warp speed, warp drive and warp factor

- Warp speed https://sto.gamepedia.com/Warp_speed
- Warp drive https://en.wikipedia.org/wiki/Warp_drive#Warp_velocities
- Warp factor http://memory-alpha.wikia.com/wiki/Warp_factor

What is the warp factor of 1,922,865,644 times the speed of light? Can the Enterprise travel that fast?

Fantasy #3. Imagine: A frog is sitting on the lily pad at *time* = 0 second. The frog doubles in size (length, width, height) each time the lily pad doubles in size (diameter).

- At what time in seconds is the frog larger than T-Rex?
- At what time in seconds is the frog larger than Godzilla?
- At what time in seconds is the frog large enough to swallow Earth?
- At what time in seconds is the frog large enough to swallow the Solar System?
- At what time in seconds is the frog large enough to swallow the Milky Way Galaxy?

Fantasy #4. Assume: The lily pad is approximately circular and the length of its diameter doubles every second. Complete Table 03 below. Calculate the area of the lily pad in m^2 (square meters).

- Area = π (*diameter* / 2)^2 = (*diameter* / 2)^2 π

Table 03 Doubling lily pad: length of diameter in meters and area in square meters

Your task: calculate the lily pad's area at *time* = 30 seconds, 40 seconds, and later.

Jump-start: We calculated the area for *diameter* = 1 m, 1024 m, and 1,048,576 m.

Time	***Diameter***	***Area***
0 second	1 m	$(1\ m/2)^2 \pi \approx 0.7854\ m^2$
10 seconds	1024 m	$(1024\ m/2)^2 \pi \approx 823{,}550\ m^2$
20 seconds	1,048,576 m	$(1{,}048{,}576\ m/2)^2 \pi \approx 8.6355E11\ m^2$
30 seconds	1,073,741,824 m	
40 seconds	1,099,511,627,776 m	
50 seconds	1,125,899,906,842,624 m	
60 seconds	1,152,921,504,606,846,976 m	
et cetera		

The mean distance of Moon's orbit from Earth is 384,400 km (kilometers). Moon's orbit is approximately a circle. What is the area of its orbit? At what time in seconds is the lily pad's area equal to or greater than the area of Moon's orbit?

The mean distance of Earth's orbit from the Sun is 149,600,000 km (kilometers). Earth's orbit is approximately a circle. What is the area of its orbit? At what time in seconds is the lily pad's area equal to or greater than the area of Earth's orbit?

Math Notes: Earth's orbit and Moon's orbit are ellipses with very small **eccentricity**.

- Orbital eccentricity https://en.wikipedia.org/wiki/Orbital_eccentricity
- Earth's mean orbital eccentricity is 0.0167. Earth's orbit is very close to circular.
- Moon's mean orbital eccentricity is 0.0549. Moon's orbit is very close to circular.
- A circle is an ellipse with eccentricity = 0.

Imagine: That lily pad keeps on doubling for as long as it takes to answer these questions.

The Milky Way is a spiral galaxy. Milky Way https://en.wikipedia.org/wiki/Milky_Way . Its size is between 100,000 ly (light years) and 180,000 ly (light years. At what time in seconds is the lily pad's area greater than the area of the Milky Way?

Math Notes: The area of an ellipse of length L and width W is $\pi \times L \times W$, also known as πLW. In more mathematical jargon, the area of an ellipse is $\pi \times A \times B$ or πAB, where A is the length of the major axis of the ellipse and B is the length of the minor axis of the ellipse. Major axis? Minor axis?

- Ellipse https://en.wikipedia.org/wiki/Ellipse

Imagine: That lily pad keeps on merrily doubling, second after second for as long as it takes to answer this question.

How big is the universe? What is the area of the universe? At what time in seconds is the area of the lily pad equal to or greater than the area of the universe?

Aha! Eureka! Abracadabra! Alakazam! YOUR STUDENTS CAN EXPLORE THESE FAR-OUT ADVENTURES.

Begin with Universe https://en.wikipedia.org/wiki/Universe .

This is the end of our lily-pad fantasy. It was fun! We loved writing it.

Reality expands to fill the available fantasies.

Bob & George • Bob Albrecht & George Firedrake • bob@geekclan.com