Math Methods for Preservice Elementary Teachers

David Moursund

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About David Moursund, Author

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- Doctorate in mathematics (numerical analysis) from University of Wisconsin-Madison, January, 1963.
- Assistant Professor and then Associate Professor, Department of Mathematics and Computing Center (School of Engineering), Michigan State University, 1963-1967.
- Associate Professor, Department of Mathematics and Computing Center, University of Oregon, 1967-1969.
- Served six years as the first Head of the Computer Science Department at the University of Oregon, 1969-1975.
- Promoted to Full Professor, University of Oregon, 1976.

* Retired in 2002, with the last 20 years of his service to the UO being in the College of Education. After retirement, worked 1/3 time in the UO College of Education 2002-2007.

- In 1974, started *The Computing Teacher,* the publication that eventually became *Learning and Leading with Technology*, the flagship publication of the International Society for Technology in Education (ISTE).
- Served on the Board of Directors of the Math Learning Center (MLC) since the MLC's inception in 1977.
- In 1979, founded the International Society for Technology in Education (ISTE). Headed this organization for 19 years.
- In 2007, founded the Information Age Education (IAE), an Oregon non-profit organization. IAE works to improve the informal and formal education of people of all ages throughout the world. See http://IAE-Pedia.org and http://I-A-E.org/.
- Author or co-author of about 65 books and several hundred articles in the fields of computers in education and math education. About 40 of the books are available free online at http://iae-pedia.org/David_Moursund_Books.
- Presented over 200 workshops and many talks in the fields of computers in education and math education.
- Served as a major professor for 82 doctoral students (six in Math, the rest in Education). See http://genealogy.math.ndsu.nodak.edu/id.php?id=8415.
- For more information about David Moursund, go to http://iae-pedia.org/David_Moursund and https://en.wikipedia.org/wiki/David_Moursund.

Table of Contents

Preface

> To understand mathematics means to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems. For the higher aims about which I am now talking are some general tactics of problems—to have the right attitude for problems and to be able to attack all kinds of problems, not only very simple problems, which can be solved with the skills of the primary school, but more complicated problems of engineering, physics and so on, which will be further developed in the high school. But the foundations should be started in the primary school. And so I think an essential point in the primary school is to introduce the children to the tactics of problem solving. Not to solve this or that kind of problem, not to make just long divisions or some such thing, but to develop a general attitude for the solution of problems. (George Polya; Hungarian and American math researcher and educator; 1887-1985.)

This book has been written to read online. It contains many imbedded links to material that both helps to justify the book's content and helps to extend it. Of course, you can decide for yourself whether you want to print a copy and read it offline. However, keep in mind that it is desirable that both you and your students develop and maintain good online reading skills and learn to take advantage of readily available online resources that are relevant to what you and they are reading. (And, of course, you save some money and a part of a tree by reading online.)

Audience

The audiences are preservice and inservice elementary teachers, teachers of the Math Methods and the Math for Elementary Teachers courses, and parents who are home schooling their elementary school age children. Secondary school teachers of mathematics may also find the book quite useful.

It may seem a little strange to you that I am recommending that you share this book with both preservice and inservice elementary school teachers. This request will make more sense to you after you read the following special introduction written just for preservice and inservice elementary school teachers.

Special Note for Elementary School Preservice and Inservice Teachers

> I assume that you are interested in becoming an elementary school teacher. In most elementary schools, each teacher is responsible for a wide part of the curriculum, including math. Thus, in your elementary education program of study you are probably required to gain some knowledge and skills in both math content (for example, through a course in Mathematics for Elementary Teachers) and math teaching methods (for example, in a course called Math Methods). In many teacher education programs, the math content course is taught by Mathematics Department faculty and the math teaching methods course is taught by College of Education faculty. In some cases these courses are team taught and/or both the Math Department and the College of Education share in teaching the two courses.
>
> At this stage of your college education and elementary education program of study, you have many years of experience in being a student in courses where someone (a teacher, faculty member, detailed syllabus, or "standards") says what you are to learn and how you are to demonstrate what you have learned.
>
> Eventually, you will become a professional in an honored profession. As a professional you will be engaged in a lifelong process of learning on the job, maintaining and increasing your professional competence. In this process you will make decisions for yourself about what you need/want to learn, how you will go about gaining relevant knowledge and skills, and how you will use your new knowledge and skills.
>
> Ask yourself the following question, "To what extent am I already an independent, self-sufficient, intrinsically motivated learner who is responsible for my own education?" A related question is, "To what extent do I want my future students to become independent, self-sufficient, intrinsically motivated learners?"
>
> My belief is that our current educational system is weak in preparing students to become lifelong learners. Young students are especially inquisitive and curious. Every teacher has an opportunity to help their students develop habits of mind that will last a lifetime. Cultivate these habits of mind in yourself, and learn to cultivate them in your students. In the future, you will thank yourself for doing so, and your students will thank you.

Thank you for reading the subsection given above. We now continue with the general introduction to this short book.

If you skipped over the quote from George Polya given above, please read it. It summarizes the “guts” of math education. Learning and teaching problem solving has been (and continues to be) a unifying theme in my professional career. I have written and presented extensively about problem solving—not only in math, but across the curriculum.

My doctorate is in mathematics. My teaching and writing experience is in a combination of math education and computers in education. I have also written extensively on the topic of brain science in education. This book summarizes some of my insights into learning and teaching problem solving from a combined math. computer science, and brain science point of view.

This book has three main goals:

1. To share some of the insights that researchers and experienced teachers of teachers have gained through designing and teaching the Math Methods course for preservice elementary teachers.
2. To present some Information and Communication Technology (ICT) ideas that are important to teaching and learning math at the elementary school level.
3. To help improve math education.

As you teach math methods, remember that some of your students will eventually become teachers of teachers. All will do this informally in their conversations with fellow teachers. Some will eventually become workshop leaders, make conference presentations, and/or teach a Math Methods course. So, teach your students in a manner that role models behaviors you want them to use with their future students and fellow teachers.

The Challenge to You and Your Students

In my opinion, the preservice elementary education Math Methods course is near the top of the list of the most important and most challenging education courses to teach. In some sense, our whole math education system rests in the hands of the few thousand faculty members who regularly teach this Math Methods course.

Preservice elementary school teachers enter a Math Methods course with widely varying math backgrounds and interest in math. Many profess to hate math and claim, "I can't do math." Some have taken the minimum math requirements to graduate from high school, and the minimum math content courses in college to meet requirements for entry into a teacher education program.

On the other hand, many students in a Math Methods course are relatively strong in math and have a good self-image of their ability to learn and use math. They may have taken a strong math program in high school, and they may have taken a year-long college sequence in discrete mathematics or calculus, and perhaps still more, in college.

The students in the Math Methods course have typically completed some version of Mathematics for Elementary Teachers coursework from a math department. The math prerequisite and the number of credits in this course or course sequence vary in different programs of study throughout the country. A relatively strong course might have college algebra or equivalent as a prerequisite and be a year sequence, three or four credits per term.

Nowadays, many teacher education programs of study require their students to demonstrate some Information and Communication Technology (ICT) knowledge and skills, and/or take an ICT course. This course might be mainly ICT content or a mixture of ICT content and ICT methods. ICT methods are now important to all teachers, since ICT has become a routine component of the informal and formal education of most students.

Chapter 1

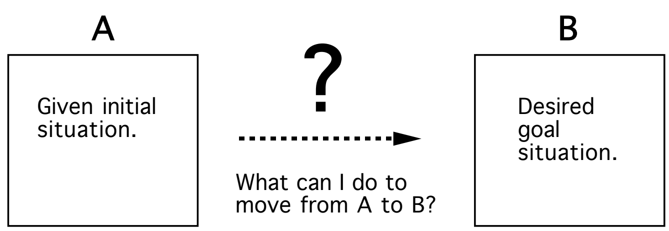
Problem Solving

> We can't solve problems by using the same kind of thinking we used when we created them. (Albert Einstein; German-born theoretical physicist and 1921 Nobel Prize winner; 1879–1955.)

At one level, we all know what problem solving is, and we do it throughout our daily lives. However, at another level, few people can give a good definition of the term *problem* and provide some useful general ideas about what the *task of problem solving* entails. This chapter provides a follow-up to the quotation from George Polya given at the beginning of the book.

What Is Problem Solving?

Problem solving consists of moving from a given initial situation to a desired goal situation. That is, problem solving is the process of designing and carrying out a set of steps to reach a goal. The diagram given below graphically represents the concept of problem solving. Usually the term *problem* is used to refer to a situation where it is not immediately obvious how to reach the goal. An identical situation can be a problem for one person and not a problem (perhaps just a simple activity or routine exercise) for another person.



Problem-solving–how to achieve the final goal.

In this book, I use the term *problem solving* to include all of the following activities:

- posing, recognizing, clarifying, and answering questions,
- posing, recognizing, clarifying, and solving problems,
- posing, recognizing, clarifying, and accomplishing tasks,
- posing, recognizing, clarifying, and making decisions, and
- using higher-order, critical, and wise thinking to do all of the above.

This broad definition is intended to encompass the critical thinking and higher-order thinking activities in every discipline. An artist, mathematician, musician, poet, and scientist all do problem solving. As you help your students learn about teaching and learning problem solving in math, I believe you should also be teaching them about problem solving across the disciplines.

Here is a formal definition of the term *problem*. You (personally) have a problem if the following four conditions are satisfied:

1. You have a clearly defined **given initial situation**.
2. You have a clearly defined **goal** (a desired end situation). Some writers talk about having multiple goals in a problem. However, such a multiple goal situation can be broken down into a number of single goal problems.
3. You have a clearly defined set of **resources** that may be applicable in helping you move from the given initial situation to the desired goal situation. There may be specified limitations on resources, such as rules, regulations, and guidelines for what you are allowed to do in attempting to solve a problem.
4. You have some **ownership**—you are committed to using some of your own resources, such as your knowledge, skills, and energies, to achieve the desired final goal.

These four components of a well defined (clearly defined) *problem* are summarized by the four words: *givens, goal, resources,* and *ownership.* If one or more of these components is less than fully filled, you have an *ill defined problem situation* (frequently called a *problem situation* or an *ill defined problem*) rather than a *well defined problem.* An important aspect of problem solving is realizing when you are dealing with an *ill defined problem situation* and working to transform it into a *well defined problem.*

Consider some *problem situations* you frequently hear about such as sustainability, global warming, globalization of business, terrorism, homelessness, drugs, and the U.S. scoring below some other countries in international tests. **For you, all these are *problem situations* until you are given sufficient information so that the *givens, goals,* and *resources* are clearly specified. In addition, you may or may not assume some *ownership* in terms of caring about specific or more general instances that pertain to these *problem situations.***

Notice also that each of the above-mentioned *problem situations* has a short name. It is easy to memorize a term such as sustainability or global warming. However, the terms mean different things to different people.

Note that we have not tried to give a careful definition of what might be meant by the world, a nation, a state, or a city to have a problem. Part of the difficulty of getting large-scale or worldwide collaboration in attempting to deal with large-scale problem situations is to get wide-scale agreement on the actual problem being addressed and who is taking ownership.

Nothing in the definition of *problem* suggests how difficult or challenging a particular problem might be for you. Perhaps you and a friend are faced by the same problem. The problem might be very easy for you to solve and very difficult for your friend to solve, or vice versa. Through education and experience, a problem that was difficult for you to solve may later become quite easy for you to solve. Indeed, it may become so easy and routine that you no longer consider it to be a problem.

People are often confused by *resources* (number 3) in the definition above. Resources merely tell you what you are allowed to do and/or use in solving the problem. Indeed, often the specification of resources is implied rather than made explicit. Typically, you can draw on your full range of knowledge and skills while working to solve a problem.

However, the rules are different when taking a test and in many other situations such as playing a game. When taking a test or doing a homework assignment, you are not allowed to cheat, steal, or plagiarize. Some tests are open book, and others are closed book. Thus, an open book is a resource in solving some test problems, but is cheating when not allowed (a limitation on resources) in solving other test problems.

Nowadays, people often have access to computers as they work to solve a problem. They draw upon both the capabilities of their own mind/brain and of Information and Communication Technology (ICT) systems. Note that this creates an educational assessment problem situation. Should students be allowed to use calculators and/or computers when taking tests? Think about possible goals of the assessment and various people who may or may not have ownership of a particular goal. I suppose I would call this type of problem situation a "sticky wicket."

Resources do not tell you how to solve a problem. For example, you decide to create an ad campaign to increase the sales by at least 20% for a set of products that your company produces. The campaign is to be completed in three months, and it is not to exceed $40,000 in cost. Three months is a time resource and $40,000 is a money resource. You can use the resources in solving the problem, but the resources do not tell you how to solve the problem. Indeed, the problem might not be solvable.

Problems do not exist in the abstract. They exist only when there is *ownership* of the problem. The owner might be a person, a group of people such as the students in a class, or it might be an organization or a country. A person may have ownership assigned by his/her supervisor in a company. That is, the company or the supervisor has ownership, and assigns it to an employee or group of employees.

The idea of ownership can be confusing. In this book, we are focusing on you, personally, having a problem—you, personally, having ownership. That is quite a bit different than saying that our educational system has an achievement problem, our country has a large-scale poverty problem, and the world has a sustainability problem.

The idea of ownership is particularly important in teaching. If a student creates or helps create the problems to be solved, there is increased chance that the student will have a sense of ownership. Such ownership contributes to intrinsic motivation—a willingness to commit one's time and energies to solving the problem. All teachers know that intrinsic motivation is a powerful aid to student learning and success.

The type of ownership that comes from a student developing or accepting a problem that he/she really wants to solve is quite a bit different from the type of ownership that often occurs in school settings. When faced by a problem presented or assigned by the teacher or the textbook, a student may well translate this into, "My problem is to do the assignment and get a good grade. I have little interest in the problem presented by the teacher or the textbook." A skilled teacher will help students to encounter challenging problems that the students really care about.

George Polya's General Problem-Solving Strategy

Quoting from George Polya:

> George Polya (1887-1985) was a Hungarian who immigrated to the United States in 1940. His major contribution is for his work in problem solving.
>
> Growing up he was very frustrated with the practice of having to regularly memorize information. He was an excellent problem solver. Early on his uncle tried to convince him to go into the mathematics field but he wanted to study law like his late father had. After a time at law school he became bored with all the legal technicalities he had to

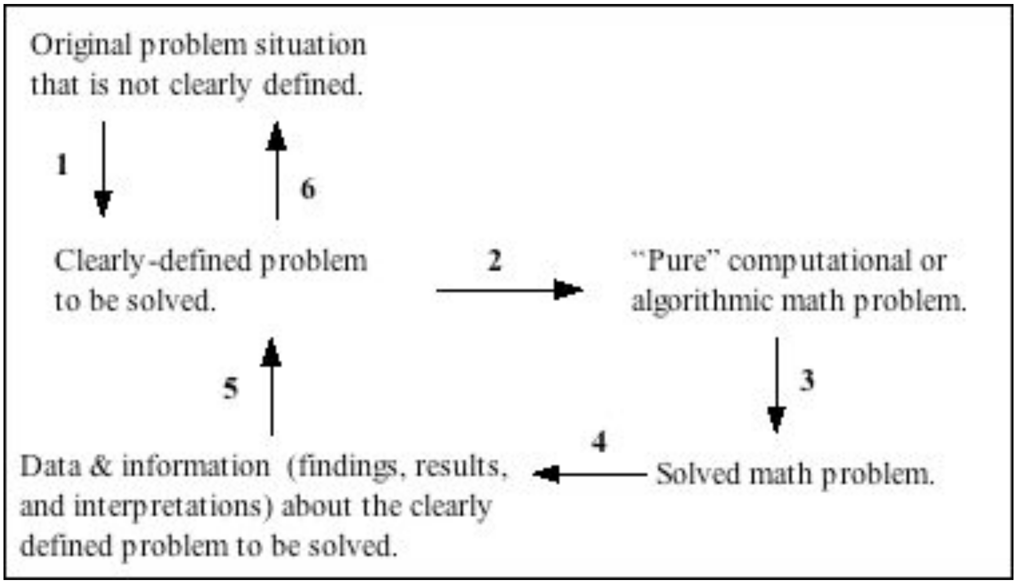
memorize. He tired of that and switched to Biology and the again switched to Latin and Literature, finally graduating with a degree. Yet, he tired of that quickly and went back to school and took math and physics. He found he loved math.

George Polya's six-step problem-solving strategy is useful in math and in most other disciplines. The following version of this strategy has been modified to be applicable in many problem-solving domains. All students can benefit from learning and understanding this strategy and practicing its use for a wide range of problems.

1. **Understand the problem.** Among other things, this includes working toward having a clearly defined problem. You need an initial understanding of the Givens, Goal, Resources, and Ownership. This requires knowledge of the domain(s) of the problem, which could well be interdisciplinary.
2. **Determine a plan of action.** This is a thinking activity. What strategies will you apply? What resources will you use, how will you use them, and in what order will you use them? Are the resources adequate to the task?
3. **Think carefully about possible consequences** of carrying out your plan of action. Place major emphasis on trying to anticipate undesirable outcomes. What new problems will be created? You may decide to stop working on the problem or return to step 1 because of this thinking.
4. **Carry out your plan of action** in a reflective, thoughtful manner. This thinking may lead you to the conclusion that you need to return to one of the earlier steps. Note that reflective thinking leads to increased expertise.
5. **Check to see if the desired goal has been achieved** by carrying out your plan of action. Then do one of the following:
    a. If the problem has been solved, go to step 6.
    b. If the problem has not been solved and you are willing to devote more time and energy to it, make use of the knowledge and experience you have gained as you return to step 1 or step 2.
    c. Make a decision to stop working on the problem. This might be a temporary or a permanent decision. Keep in mind that the problem you are working on may not be solvable, or it may be beyond your current capabilities and resources.
6. **Do a careful analysis** of the steps you have carried out and the results you have achieved to see if you have created new, additional problems that need to be addressed. Reflect on what you have learned by solving the problem. Think about how your increased knowledge and skills can be used in other problem-solving situations. Work to increase your reflective intelligence!

A Diagram Representing Applying Polya's Method to a Math-related Problem

The diagram below captures the essence of the six-step Polya process in a math problem-solving situation. I like to use this diagram when I talk about math problem solving. It can be thought of as "shorthand" for the Polya six-step method, as a way to introduce math modeling, and as a way to raise the issue of the roles of computers in solving math problems. Note also that Polya's six-step method and the diagram are *heuristics* rather than *algorithms.* In simple terms, an *algorithm* is a finite step-by-step method that is guaranteed to solve a particular type of problem. A *heuristic* is like an algorithm, but it is not guaranteed to solve a particular type of problem.



1. Problem posing and problem recognition to produce a Clearly Defined Problem;
2. Mathematical modeling;
3. Using a combination of careful thinking, and computational or algorithmic procedures to solve a computational or algorithmic math problem;
4. Mathematical "unmodeling";
5. Thinking about the results to see if the Clearly Defined Problem has been solved; and
6. Thinking about whether the original Problem Situation has been resolved. Steps 5 and 6 also involve thinking about related problems and problem situations that one might want to address or that are created by the process of attempting to solve the original Clearly-Defined Problem or resolve the original Problem Situation.

In steps 1 and 2, a person works to understand a problem situation and makes a decision as to whether it might be useful to attempt to solve the problem using math. A person deciding to take a math-oriented approach to resolving the problem situation attempts to represent or model the problem situation using the language of mathematics. This math modeling leads to having a math problem that may or may not be solvable, and that may or may not be solvable by the person attempting to solve the problem.

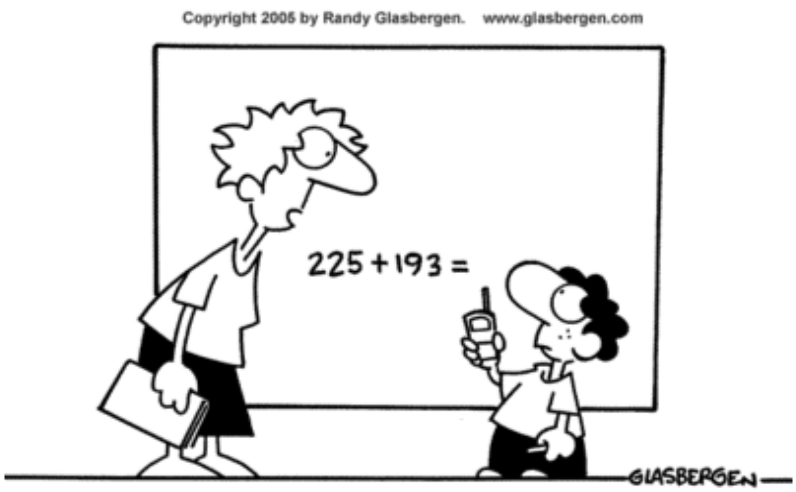
The great majority of K-12 math education is focused on students learning to do step 3 using paper and pencil algorithms, yet step 3 is what calculators and computers are best at. Thus, the great majority of math education at the K-12 levels is spent helping students learn to compete with calculators and computers in areas that are not well suited to the capabilities of a human mind but that are well suited to computers. (Actually, I make this assertion without adequate research evidence to back it up. My informal research comes from information provided to me by large numbers of students I have taught. They provided estimates in the 70% to 75% range.)

In step 6, the person who has a solution to the math problem extracted when dealing with step 1 checks the degree to which the results achieved are relevant to the original problem situation and decides whether the overall process has been useful in trying to resolve the original problem situation.

Chapter 2

Some Introductory Ideas

This chapter presents some ideas that I believe are important content to weave into a Math Methods course. Each person teaching such a course has their own ideas, so please do not take mine as "musts" in your teaching.



"You have to solve this problem by yourself. You can't call tech support."

The disciplines of Computer Science and Mathematics are closely linked. This suggests that math teachers at every grade level may (could, should, likely will) have some responsibilities to help their students learn some math-related ICT and Computer Science. Of course, the math content, interests, and needs, as well as the ICT content, interests, and needs of students in a Math Methods course vary tremendously across the nation.

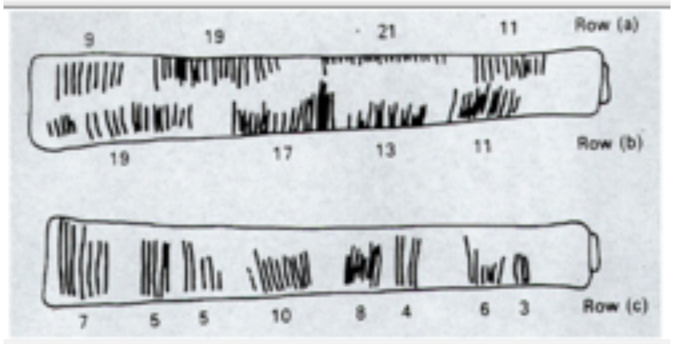
The teachers of Math Methods courses sometimes complain about the modest math and ICT content knowledge of their students. They want to spend the valuable and limited course time teaching Math Methods, not math and computer content.

This ongoing problem creates tension between teachers of Math Methods courses and teachers of math and computing content courses. In some cases, there is close cooperation among the departments and faculty who teach the content and methods courses. The same faculty members may teach both courses, or team teaching may occur. This can be especially effective in a collaboration between faculty who teach Math for Elementary Teachers and Math Methods. Both courses need to stress math content, math pedagogy, and math pedagogical content knowledge.

A Tidbit of Math Education History

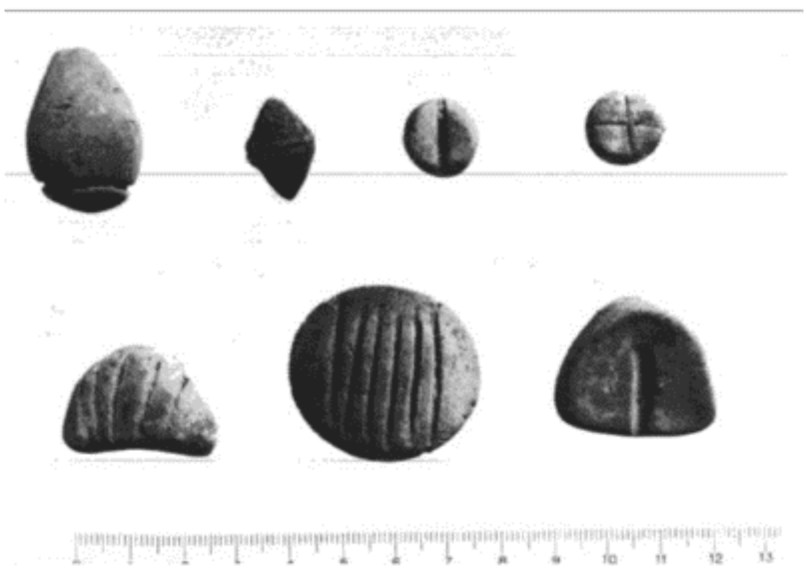
Mathematics as a formal area of teaching and learning was developed about 5,000 years ago by the Sumerians. They did this at the same time as they developed reading and writing. However, the roots of mathematics go back much more than 5,000 years.

Throughout their history, humans have faced the need to measure and communicate about time, quantity, and distance. The Ishango Bone is a bone tool handle approximately 20,000 years old.



Ishango bone from approximately 20,000 years ago.

The picture given below shows Sumerian clay tokens whose use began about 11,000 years ago (see http://www.sumerian.org/tokens.htm). Such clay tokens were a predecessor to reading, writing, and mathematics.



Clay tokens from about 11,000 years ago.

The development of reading, writing, and formal mathematics 5,000 years ago allowed the codification of math knowledge, the development of formal instruction in mathematics, and also began a steady accumulation of mathematical knowledge.

What Is Mathematics?

Students in a Math Methods course have quite varying insights into the *What is mathematics?* question. As a teacher in such a course, you might want to have your students do an in-class discussion activity on that topic and/or some reading about it. For example, see the *IAE-pedia* article, What is Mathematics?

Mathematics and the Study of Patterns

A common answer to *What is mathematics?* is to say that math is the study of *patterns*. That is correct, but this same definition applies to many other disciplines. A fundamental aspect of how our brain works is that it does so by pattern matching. Let me give you a brain science example that will help fix this idea in your mind.

Someone is talking to you. That person's mind/brain and body create sound patterns that are representative of idea the person wants to communicate to you. Hmm. How is an idea stored in a person's brain? This is a "deep" question in brain science research. A brief answer is that it is stored as a collection of connections between neurons. Human memory is not like written words stored on a page or binary coded words stored in a computer memory.

So, the person speaking to you has an idea, and the person's brain translates this into a sequence of words that are stored as patterns in that person's brain. The person's brain sends appropriate messages to various body parts that move air toward the mouth, and move mouth, lips, and tongue in appropriate manners. The stored sentence comes out as a sequence of sound waves. (As an aside, as you think about the complexity of this process and that the speaker is hearing the results, it is no wonder that speech is punctuated by so many "you knows" and other types of pauses.)

The pattern of sound waves moves through the air, perhaps being somewhat messed up by other sound waves from other sources, such as background conversation of others, background music, the hum of an air conditioner or furnace, and so on.

The sound waves of the sentence and the interference hit your ear and are translated by your ear mechanism into electro-chemical nerve signals that store the information as memory patterns in your short-term auditory memory. Your brain then processes this short-term memory pattern and figures out what this sentence means. Note that it is possible to merely memorize the sounds that have been spoken without attaching any meaning to them. Attaching meaning requires matching the received patterns with stored patterns (data, information, knowledge) in your brain. Your understanding of an utterance depends on what you already know. This, of course, relates to the constructivist theory of learning, in which a person builds knowledge and understanding upon existing knowledge and understanding. Constructivism is discussed in chapter 4.

Finally, imagine that you are a student and the person talking to you is communicating in the language of mathematics and drawing a math diagram on the white board at the same time. In some sense, the math communication is in a language that is foreign to you. The addition of a diagram to the conversation means that your visual system has to deal with a pattern of light waves and…

In summary, I don't think that it helps a student much to tell the student that math is the study of patterns. Rather, emphasize that finding and using patterns is essential in every discipline of study. Each discipline has some patterns unique to the discipline, so learning to recognize these patterns and make use of them is an important aspect of learning a discipline. Since math is so useful across the disciplines, it is important for math students to learn to recognize math patterns not only in the math they study, but also in other disciplines they study.

Mathematics Is a Discipline of Study

A discipline (an organized, formal field of study) such as mathematics tends to be defined by the types of problems it addresses, the methods it uses to address these problems, and the results it has achieved. **Problem solving is a unifying concept in every discipline of study**.

Here are several ideas to keep in mind as you teach and learn math:

1. **Mathematics as a human endeavor.** For example, consider the math of measurement of time such as years, seasons, months, weeks, days, and so on. Or, consider the measurement of distance, and the different systems of distance measurement that developed throughout the world. Or, think about math in art, dance, and music. There is a rich history of human development of mathematics and mathematical uses in our modern society.
2. **Mathematics as a vibrant discipline that has considerable breadth and depth, and a long history.** Nowadays, a Ph.D. research dissertation in mathematics is typically focused on definitions, theorems, and proofs related to a single problem in a narrow sub-field in mathematics.
3. **Mathematics as an interdisciplinary language and tool.** Like reading and writing, math is an important component of learning and "doing" (using one's knowledge) in each academic discipline. Mathematics is such a useful language and tool that it is considered one of the "basics" in our formal educational system.
4. **Definitions and proofs are core (foundational) to mathematics.** Mathematicians often talk about the beauty of a particular proof or mathematical result.

Consider the task of developing a K-8 or K-12 mathematics curriculum. How much emphasis should be given to each of the items 1-4 above, and what specific content should be taught?

An educational system that overemphasizes the past tends to be out of tune with younger people. An educational system that overemphasizes the future tends to be out of tune with older people. Thus, our educational system faces a continuing challenge of seeking an appropriate balance between the past and the future.

The challenge becomes greater in times of rapid change. Today we live at a time of rapidly increasing technologically-based change. The world is growing "smaller." The world as a whole faces the challenge of sustainability, climate change, increasing population, diseases that spread rapidly, large numbers of hungry people living in poverty, and so on. Ask yourself: "In what ways does knowledge and skill in math affect a person's understanding of such problems and help in dealing with such problems?"

Here are a few rapidly changing areas to think about:

- Information and Communication Technology (ICT), including computers, robots, artificial intelligence, telecommunications, and the Internet.
- Genetics and genomes, including cognitive neuroscience (brain science) and genetic engineering.
- Nanotechnology—the technology of the very, very small.
- Big data. Not only is "big brother" watching you, big data is a powerful new aid to research, development, marketing, and so on.

Math is quite important in each of these areas! Thus, our math education system faces the ongoing challenge of helping to prepare students for life in a world where math is both an important discipline in its own right and is also an important part of our changing world. This challenge is becoming greater as the pace of technological change in our world continues to increase.

George Polya Revisited

Please read once more the quote from George Polya given at the beginning of this book. Math is an important discipline of study because it is a powerful aid to representing and solving problems. The math-related problems that a person encounters during their lifetime range from those that the person encounters over and over again to problems that the person finds somewhat novel to problems that are completely new to the person. No matter how much math a person learns, they are still faced by this range of math-related problems. A good math education curriculum prepares a student to deal with a range of math-related problems that are appropriate to the abilities, interests, and needs of the student.

The discipline of math is also growing because of its use and importance in many other disciplines—and these are rapidly growing and changing.

I think you can see the challenges the above ideas bring to you, as a Math Methods teacher. Of course, these same challenges will face your students as they become teachers. Thus, you should give careful thought to how you will help to prepare your students for their futures as teachers of math and other disciplines.

You might want to use this challenge as a topic for a writing assignment, small group discussions, and whole class discussion. Exploration of this topic might also include investigation of the Math Education Wars discussed below.

Improving Math Education

An underlying and unifying theme in this book is that our math education system can be substantially improved. Here are four general approaches often used in working to improve our math education system:

1. Identify students with learning challenges related to learning and doing math, and provide them with special help. Dyslexia (a reading problem), dyscalculia (an arithmetic calculation problem), and math anxiety are examples. Here is a question that you might want to use in small group or whole class discussions: What is math anxiety, and do you and/or someone you know have math anxiety?
2. Do what we have been doing in the past, but do it better. This includes increasing the number of minutes a day that students study math, imposing more required math courses, setting higher standards, and use of high-stakes tests. Back to basics, "Algebra for all," and high-stakes algebra testing as a requirement for high school graduation are some examples of this movement.
3. Draw upon past and ongoing math educational research and on progress in areas such as ICT and brain science to implement significant research-based changes in the content, pedagogy, and assessment in math education. Place much more emphasis on learning for understanding, higher-order thinking, and problem solving—and less emphasis on rote memory. Recognize and make use of the fact that, increasingly, computerized tools have

built-in artificial intelligence so that they can serve both as a tool and as a teacher of a person learning to use the tool. See chapter 4 of my book, *Technology and Problem Solving: PreK-12 Education for Adult Life, Careers, and Further Education.*

Moursund, D. (2/28 2015). *Technology and Problem Solving: PreK-12 Education for Adult Life, Careers, and Further Education.* Eugene, OR: Information Age Education. Microsoft Word: http://i-a-e.org/downloads/free-ebooks-by-dave-moursund/266-technology-and-problem-solving-in-prek-12-education.html. PDF: http://i-a-e.org/downloads/free-ebooks-by-dave-moursund/267-technology-and-problem-solving-in-prek-12-education-1.html.Web: http://iae-pedia.org/Technology_and_Problem_Solving.

4. Design and implement technology-based aids to learning education. As computer technology continues to improve, it becomes increasingly possible to provide individualization of instruction via use of highly interactive, intelligent computer-assisted learning (HIICAL). Some of the process of translating research theory and results into practice can be significantly aided by the use of computer technology.

Part of the Math Education Wars controversy is between those who favor the second approach described above and those who favor the third approach.

The third and fourth approaches will eventually lead to a considerable change in the roles of classroom teachers. In my opinion, over the long run these last two approaches will come to dominate the first two.

Teaching the Way You Were Taught

There is substantial evidence that teachers teach the way they were taught. What are your thoughts when you read such an assertion and it is not backed up by appropriate references to research or credible, high-level expertise? What do your students think about such as assertion?

When I am writing and an assertion of this sort comes to my mind (and, to my fingers), I often spend time on the Web, trying to find evidence to support the assertion. I think that this is a habit that you (as a teacher of teachers) should be cultivating in your preservice teachers.

My 2/18/2016 Google search of the expression *teachers teach the way they were taught* produced nearly 72 million results. That large number suggests this is a topic that many people have written about. It does not tell me if my assertion is correct.

Google relies on a proprietary ranking process that involves perhaps 200 or more criteria. I started at the beginning of their sorted list, looking for results that might serve my needs.

Remember, I am not a researcher in this area, and I don't need to do a careful search of the literature to find the very latest research, or what aspects of this topic I might want to do empirical research on in order to advance the field. What I need to find out is whether my assertion is very likely a correct assertion and to provide some useful information to my readers.

Near the top of this list I found the article:

Kennedy, M.M. (1999). The role of preservice teacher education. Retrieved 2/18/2016 from https://www.msu.edu/user/mkennedy/publications/docs/Teacher%20Ed/RoleofTE-LDH/Kennedy99%20Role%20of%20TE.pdf.

Kennedy's article is chapter 3 from the book:

> Darling-Hammond, L., & Sykes, G. (1999). *Teaching as the learning profession: Handbook of teaching and policy.* San Francisco: Jossey Bass.

Aha! Although the book is quite old, I recognize the name Linda Darling-Hammond, a world-class math educator. So, I went to this chapter and started reading. Wow! Just what I was looking for. Before delving into the chapter too deeply, however, I used the Web to check on the professional qualifications of Mary M. Kennedy. I was amused to see she finished her undergraduate studies in education at Michigan State University during the same time that I was teaching in the math department there. She has great credentials.

The chapter begins with the assertion that, in the United States, most non-teachers and teachers subscribe to a *received wisdom* theory of how students should be taught, but most teachers of teachers do not believe in this theory. Quoting from the chapter:

> According to *received wisdom*, teaching is fundamentally a self-evident practice. What to teach should be obvious if you know your subject, and what to do at any given moment should be obvious from the situation. Therefore learning to teach consists of two main parts:
>
> - you learn the subject you intend to teach through college-level liberal arts courses, and
> - you refine your technique and personal style through experience in your own classroom.
>
> Most versions of the received wisdom end here. Some versions add a small role for teacher education, acknowledging that there might be some benefit from studying child psychology or perhaps research on teaching. **But the role of teacher education is still considered to be relatively modest.**
>
> Even reform movements, which usually acknowledge that there might be more to teacher learning than meets the eye, often subscribe to the received wisdom and concentrate more attention on continuing professional development than on preservice teacher education. Consistent with received wisdom, reformers tend to believe that a great deal of teacher learning occurs in the context of practice, that teachers can continue to refine their techniques throughout their careers, and that this is where reform efforts should be concentrated. [Bold added for emphasis.]

My goodness! Reread the bolded sentence. Teacher education has been under attack for a long time. I wonder why so many non-educators and teachers are so "down" on teacher education? Here is another quote from the Kennedy chapter:

> The sociologist Dan Lortie pointed out over twenty years ago that teachers go through a lengthy apprenticeship of observation in that they spend their entire childhoods observing teachers teach. **Lortie suggested that the endurance of traditional teaching practice derives in part from the fact that teachers are highly likely to teach in the way they themselves were taught.** Their experiences in primary and secondary schools give them ideas about what school subject matter is like, how students are supposed to act in school, and how teachers are supposed to act in school. Thus, when they begin to teach, they adopt the practices of their former teachers. If their elementary teachers represented the school subject of writing as a set of grammar rules, for instance, rather than as a way to organize thoughts and communicate ideas, they will tend to teach writing this way themselves. [Bold added for emphasis.]

Let's use PreK-8 school teachers who teach math as an example. These teachers first learned about how to teach math when they were very young children, learning from their parents, caregivers, and perhaps from older siblings. The math pedagogy education of these preservice teachers continued as they progressed through an elementary school and middle school math program of study. Still further continuation was provided by what they observed as they studied math in high school.

Thus, the typical preservice elementary school teacher knows a great deal about how math is taught before he or she takes any education courses, a Math for Elementary Teachers sequence, or a Math Methods course. Teachers of preservice and inservice teachers know that it can be difficult to change this predisposition. Teachers tend to teach the way they were taught, and parents tend to expect that their children will be taught in the way that they were taught.

The use of calculators and computers in math classes provides an excellent example of resistance to change. The National Council of Teachers of Mathematics has supported the use of calculators in elementary school math education since 1980. Even now, more than 35 years later, a great many preservice and inservice math teachers make and implement their own decisions about whether calculators are appropriate or not. Thus, to a considerable extent, they **do teach** about and allow use of calculators, or they **do not teach** about and allow use of calculators, based mainly on their personal opinions. Gradually, the use of calculators has been allowed in some high-stakes assessments. However, to a large extent people designing such assessments do so in a manner that a calculator is of modest use.

It seems strange to me that after all of these years we still lack convincing research evidence on effective methods for making use of calculators in K-8 math education, effective methods of preparing teachers to do this teaching, and on the long-term effectiveness of these methods.

> As an "aside," when I first started recommending use of calculators in elementary school math education, I worried a little about whether this might influence the number of students who eventually would complete doctorates in mathematics. Perhaps the early use of calculators would warp or shape their minds so that they became less capable of learning more advanced mathematics. These concerns point out one of the difficulties in educational research. For the most part, educational researchers are not able to conduct studies that track the effects of an educational change over a decade or several decades before the change is widely implemented.

Some of the early work in computer-assisted instruction (CAI) in school mathematics was done more than 55 years ago. Since then computers have become much less expensive, the theory of CAI has made considerable progress, the quality of CAI materials has been substantially improved, and there has been a very large amount of research on the use of and effectiveness of CAI in K-12 math instruction.

So, paralleling my question about the use of calculators, I also ask if the cost-effectiveness of math CAI in K-8 education is such that these computer aids to teaching and learning math should now be a routine component of math education. I strongly believe the answer is "yes." And if the answer is "yes," shouldn't modern Math Methods courses ensure that all preservice K-8 teachers have a high level of knowledge and skill in making use of this technology? **It is not easy to work effectively with a classroom full of students, each using a computing device, and each at a different place in a lesson and/or in different lessons.**

Math Pedagogical Content Knowledge

The importance of pedagogical content knowledge (PCK) in teaching is a relatively new idea pioneered by Lee Shulman. The basic concept is that good teaching involves knowledge of the content area being taught, general knowledge of how to manage and teach students, and specific knowledge about how to teach the content of the specific discipline being taught.

As a "far out" example, consider the challenge a research mathematician would face if asked to teach elementary school children about the number line. The research mathematician has a huge amount of math content knowledge in this area. This includes ideas about different sizes of infinity, rational, irrational, and transcendental numbers, repeating decimals, numbers to different bases, number theory, and so on. The math researcher can likely give a rigorous proof that (-3) x (-5) = (+15). But, how will this go over with students at various grade levels? What should be taught, how should it be taught, and how does one assess the results?

How does one explain the number line to a learning disabled second grader, an average second grader, or a talented and gifted second grader? What will each already know, and how does one help these varying students build new knowledge and understanding onto what they already know?

The students are both building their math models, and building upon their mental models of the number line. Many will eventually develop a useful visual model of the number line. How might a bead frame (or abacus), graph paper, or a variety of math manipulatives help in each case? What virtual manipulatives available free on the Web might help? What does one do for students who are inherently poor in spatial visualization and have difficulty "seeing" things in their mind's eye?

The importance of math PCK was researched in Liping Ma's doctoral dissertation and widely disseminated via her book:

> Ma, Liping (1999). *Knowing and Teaching Elementary Mathematics.* Mahwah, NJ: Lawrence Erlbaum.

Ma's doctoral research compared elementary school math education in the United States with that in China where she grew up. She makes it clear how math content knowledge and math pedagogical content knowledge of teachers differ between the U.S. and China. See Vikas Bajaj's 12/13/2013 interview with Liping.

Math Manipulatives

The previous section mentioned math manipulatives. Children learn to count on (using) their fingers, which can be thought of as a type of math manipulative. Recent research strongly supports use of this math manipulative.

> Boaler, J., Chen, L., Williams, C., & Cordero, M. (2016). Seeing as understanding: The importance of visual mathematics for our brain and Learning. *youcubed at Stanford University.* Retrieved 4/21/2016 from https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/2016/04/Visual-Math-Paper-vF.pdf.

Quoting from this article:

> Good mathematics teachers typically use visuals, manipulatives and motion to enhance students' understanding of mathematical concepts, and the US national organizations for mathematics, such as the National Council for the Teaching of Mathematics (NCTM) and

> the Mathematical Association of America (MAA) have long advocated for the use of multiple representations in students' learning of mathematics.
>
> But for millions of students in US mathematics classes, mathematics is presented as an almost entirely numeric and symbolic subject, with a multitude of missed opportunities to develop visual understandings.
>
> Students who display a preference for visual thinking are often labeled as having special educational needs in schools, and many young children hide their counting on fingers, as they have been led to believe that finger counting is babyish or just wrong. This short paper, a collaboration between a neuroscientist and mathematics educators, shares stunning new evidence from the science of the brain, showing the necessity and importance of visual thinking – and, interestingly, finger representations—to all levels of mathematics.
>
> Evidence from both behavioral and neuroscience studies shows that when people receive training on ways to perceive and represent their own fingers, they develop better representations of their fingers, which leads to higher mathematics achievement (Ladda et al., 2014; Gracia-Bafally and Noël, 2008). Researchers found that when 6 year old's improved the quality of their finger representation they improved in arithmetic knowledge, particularly subitizing1, counting and number ordering. Remarkably the 6 year old's finger representation was a better predictor of future mathematics success than their scores on tests of cognitive processing.
>
> One of the recommendations of the neuroscientists conducting these important studies is that schools focus on finger discrimination. The researchers not only point out the importance of number counting on fingers, for brain development and future mathematics success, they advocate that schools help students' discriminate between their fingers. This seems particularly significant to us given that schools pay no attention to finger discrimination now and no published curriculum that we know of encourages this kind of mathematical work.

Some people seem to view math manipulatives as a sort of toy for use by young children. There is growing realization, however, that manipulatives are a really valuable part of math education for children who are at the Concrete Operations and Pre-Operational stages of Piaget's 4-stage cognitive development theory.

Marilyn Burns is a highly respected math educator. Quoting from her article, Seven Musts for Using Manipulatives:

> You find them in classrooms across the nation—buckets of pattern blocks; trays of tiles and cubes; and collections of geoboards, tangrams, counters, and spinners. They've been touted as a way to help students learn math more easily. But many teachers still ask: Are manipulatives a fad? How do I fit them into my instruction? How often should I use them? How do I make sure students see them as learning tools, not toys? How can I communicate their value to parents? Are they useful for upper-grade students, too?
>
> I've used manipulative materials at all levels for 30 years, and I'm convinced I can't — and shouldn't — teach without them.

Now we have both physical (concrete) manipulatives and virtual (computer-based) manipulatives. They each have some advantages. Doug Clements is an outstanding math education researcher. Here is an article (a golden oldie) that I consider to be a "must read" for preservice elementary school teachers:

> Clements, D.H. (1999). 'Concrete' manipulatives, concrete ideas. *Contemporary Issues in Early Childhood.* 1(1), 45-60. Retrieved 2/8/2016 from http://www.didax.com/articles/concrete-manipulatives-concrete-ideas.cfm. Quoting from this article:
> Students who use manipulatives in their mathematics classes usually outperform those who do not, although the benefits may be slight. This benefit holds across grade level, ability level, and topic, given that use of a manipulative "makes sense" for that topic. Manipulative use also increases scores on retention and problem solving tests. Attitudes toward mathematics are improved when students have instruction with concrete materials provided by teachers knowledgeable about their use.
> However, manipulatives do not guarantee success. One study showed that classes not using manipulatives outperformed classes using manipulatives on a test of transfer [of learning]. In this study, all teachers emphasized learning with understanding. In contrast, students sometimes learn to use manipulatives only in a rote manner. They perform the correct steps, but have learned little more. For example, a student working on place value with beans and beansticks used the (one) bean as ten and the beanstick (with ten beans on it) as one.

Here is a link to a math manipulatives project that initially received federal funding in the U.S. Some of its products are available free.

> Virtual Manipulatives (n.d.). National Library of Virtual Manipulatives for Interactive Mathematics. Retrieved 2/7/2016 from http://nlvm.usu.edu/en/nav/vlibrary.html.

The Math Learning Center provides a number of free, high-quality virtual manipulatives. See their catalog. (Full disclosure: I am on their Board of Directors and I provided funding to help get this project started.)

Communicating in the Language of Mathematics

Communication lies at the very heart of education. This includes communication with oneself and communication with others. It includes communication with machines, such as computers. It includes communication over time and distance. For example, you know about Euclid and Pythagoras. They both are communicating with you over time and distance.

Teachers of math are helping their students gain increased expertise in oral and written communication in the language of mathematics. Thus, as you teach a Math Methods course you will want to role model communication in math and help your students gain an increased level in such communication and also in understanding why this is important to their future.

All preservice teachers learn about the idea of reading (or reading and writing) across the curriculum. However, most learn very little about reading and writing of math as being an important component of learning math. As you teach your Math Methods course, you may well want to make use of such strategies as having your students do journaling and regularly participate in small group and whole class discussions. You may well have them develop and/or modify some math lesson plans. You may want to insist that they read the textbook and other written material. All of these types of activities will give your students practice in communicating in math. (Many faculty find that this turns out to be a struggle. Think about giving a one or two question short quiz over the reading at the start of each class period as a lead-in to this skill.)

Here is an activity that you might want to try out with your students. Have them individually answer the question given below. Then have them discuss their answers in small groups and in a whole class debriefing.

> Think back over your own math content education. Try to remember a time in your precollege education when you were expected to learn some math by reading a math book. Assess yourself in your current level of skill in learning math by reading a math book.

The point to this activity is that, for the most part, precollege math tends to be taught by "oral tradition." That is, students are not expected to learn new math topics by reading about them in their math book, on a computer, or in written handouts.

There is a lot of good material on reading and writing in math education available on the Web. My 2/7/2016 Google search of the expression *writing in math* produced over 260 million results. Marilyn Burns has long been a world leader in this area. Here in one example of her "writing in math" resources available on the Web:

> Burns, Marilyn (2004). *Writing in Math.* Retrieved 2/7/2016 from http://www.mathsolutions.com/documents/2004_writing_in_math.pdf.

Here is another article that you might find useful:

> Wolpert-Gawron, Heather (5/1/2015). 4 Tips for Writing in the Math Classroom. *Edutopia.* Retrieved 2/7/2016 from http://www.edutopia.org/blog/four-tips-writing-math-classroom-heather-wolpert-gawron.

The *IAE-pedia* contains an extensive and quite popular document about Communication in the Language of Mathematics. Here are a few important ideas quoted from this document:

> Although one can spend a lifetime studying math and still learn only a modest part of the discipline, young children can gain a useful level of math knowledge and skill via "oral tradition" even before they begin to learn to read and write. Oral and tangible, visual communication in and about math is an important part of the discipline.
>
> Reading and writing are a major aid to accumulating information and sharing it with people alive today and those of the future. This has proven to be especially important in math, because the results of successful math research in the past are still valid today.
>
> Reading and writing (including drawing pictures and diagrams) are powerful aids to one's brain as it attempts to solve challenging math problems. Reading and writing also help to overcome the limitations of one's short-term memory.
>
> The language of mathematics is designed to facilitate very precise communication. This precise communication is helpful in examining one's own work on a problem, drawing upon the previous work of others, and in collaborating with others in attempts to solve challenging problems.
>
> Our growing understanding of brain science is contributing significantly to our understanding of how one communicates with one's self in gaining increased expertise in

solving challenging problems and accomplishing challenging tasks in math (and in other disciplines).

Information and Communication Technology (ICT) has brought new dimensions to communication, and some of these are especially important in math. Printed books and other "hard copy" storage are static storage media. They store information, but they do not process information. ICT has both storage and processing capabilities. It allows the storage and retrieval of information in an interactive medium that has some machine intelligence (artificial intelligence). Even an inexpensive handheld, solar-battery 6-function calculator illustrates this basic idea. There is a big difference between retrieving a book that explains how to solve certain types of equations and making use of a computer program that can solve all of these types of equations.

We all understand the idea of a native language speaker of a natural language. Students learning an additional language will often progress better when taught by a native language speaker who can fluently listen, read, talk, write, and think in the language, and who is skilled in teaching the language. We prefer that this teacher be fluent in a "standard" version of the language and not have a local accent and vocabulary that would give pause to many native speakers of the same language. The same idea holds in math education. The math educational system in the United States is significantly hampered because so many of the people teaching math do not have the math knowledge, skills, and math pedagogic knowledge—and, most important, level of fluency—that would classify them as being math education native language speakers.

I have bolded the last paragraph because of its importance in math education. It is essentially an argument that elementary school mathematics should be taught by a specialist in math. Some countries do this, and others don't. The U.S. is one of the countries that has only modest math requirements in the preparation of elementary school teachers who then teach math as part of their everyday teaching schedule.

This is a good topic for class discussion in a Math Methods or a Math for Elementary Teachers course.

Good Math Lesson Plans

In a Math Methods course, students often study lesson plans written by others and/or create their own sample math lesson plans. They may be provided with a template for general lesson plan writing or one specifically designed for math lesson plans.

A good example of such a math-specific lesson plan template is is available in the *IAE-pedia*. It stresses many of the ideas included in the book you are currently reading and is one of the most popular entries in the *IAE-pedia.* This *IAE-pedia* document served as a major resource in developing the following free book:

Moursund, D. (March, 2012). *Good Math Lesson Planning and Implementation.* Eugene, OR: Information Age Education. Download the PDF file from http://i-a-e.org/downloads/doc_download/230-good-math-lesson-plans.html. Download the Microsoft Word file from http://i-a-e.org/downloads/doc_download/229-good-math-lesson-plans.html.

The following is quoted from the book:

> This book is not a compendium of math lesson plans. Indeed, it contains just a very few brief examples. You can find oodles of math lesson plans in books and on the Web. …
>
> The accumulation of math lesson plans contributes to math education. However, if math education could be substantially improved by the accumulation and distribution of math lesson plans, math education would be rapidly improving. There is something missing in this "formula." What is missing are the human and the "theory into practice" components.
>
> Each learner and each teacher is unique. As teachers and as learners we are not machines. Good lesson planning and implementation reflects the human capabilities, limitations, knowledge, and experience of both the teacher and the learners.
>
> There are some aspects of teaching in which computers can out perform human teachers. We are living at a time in which computer-assisted learning and distance learning are gaining rapidly in capabilities, use, and importance. Good teachers and good teaching accommodate and make effective use of this major addition to the aids useful in teaching and learning. These newer aids, along with older aids, do not obviate the value of and need for good teachers and the need for good teachers. They do change the teacher's job. Remember, it is the teacher plus aids to the teacher that facilitate good teaching.
>
> I think of a personalized math lesson plan as an extension of a human teacher. It supplements and extends the human capabilities of a human teacher. This is a unifying idea in this book.

Problem-based and Project-based Learning

See the *IAE-pedia* article, Math Project-based Learning. The document discusses both problem-based and project-based learning, and it contains a number of examples.

Problem-based learning is often used in teaching math. Quoting from an *ERIC Digest* article, Problem-Based Learning in Mathematics:

> Problem-based Learning (PBL) describes a learning environment where problems drive the learning. That is, learning begins with a problem to be solved, and the problem is posed is such a way that students need to gain new knowledge before they can solve the problem. Rather than seeking a single correct answer, students interpret the problem, gather needed information, identify possible solutions, evaluate options, and present conclusions. Proponents of mathematical problem solving insist that students become good problem solvers by learning mathematical knowledge heuristically.

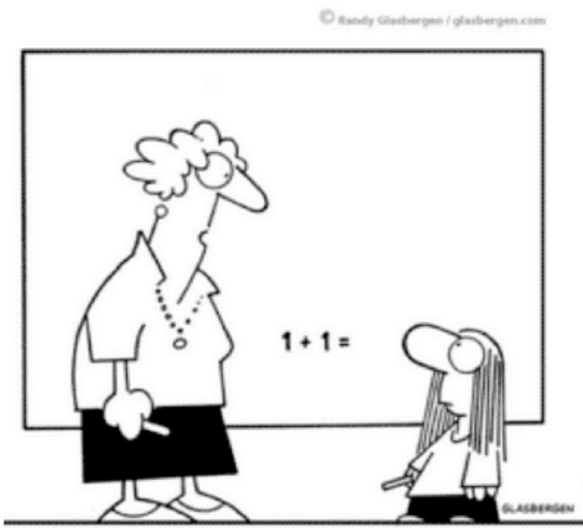
Problem-based learning tends to be "authentic" in that it can engage students in "doing" math and "using" math in working on real world and other challenging problems.

Project-based learning is less often used in math education than problem-based learning, but is a valuable part of the math education repertoire of many teachers. Project-based learning is a way to involve students in learning and using math, and is often a team activity. This helps students learn to learn mathand use math in a collaborative environment. Project-based learning is a good vehicle for helping students make progress on a number of math educational goals not directly covered in the "traditional" math curriculum. Not least among these goals is to help make math education a pleasant and rewarding discipline of study that adults will look back on with fond memories.

Chapter 3

Prerequisites: A Delicate and Challenging Issue

This chapter examines the prerequisites one might expect students in a Math Methods course to have met. It also explores some possible also the prerequisites relevant to being an effective and successful faculty member in such a course.



"Yes, this will be useful to you later in life."

"Algebra class will be important to you later in life because there's going to be a test six weeks from now."

Prerequisites for Math Methods Students

Here is an important question. In the Math Methods course that you teach, what is the math content knowledge and skills prerequisite? Many Math Methods teachers pass lightly over this delicate prerequisite situation. I strongly believe you are doing your students a disservice if you ignore the issue. And, it relates closely to the question, "How much of the very limited and very valuable time in your Math Methods course is (or should be) used in math content remediation?" I believe you should make a carefully reasoned decision about this matter.

As an aside, every math teacher at every grade level must routinely face the prerequisite issue in their day-to-day teaching. So, it is an importation topic to cover in a Math Methods course. How do you effectively work with students who lack the prerequisite knowledge and skills you believe are necessary to the lesson you are teaching?

How do you determine if each of your students meets these math content knowledge and skills prerequisites? For example, you might make use of a "self-assessment" math content test that includes links to where a student can find good review and remediation materials. A variety of free college math placement tests are available online.

You might want to raise this prerequisite issue (for your students, and for their future students) near the beginning of your course. Bring the problem out into the open, and then role model an effective way of dealing with it. Help your students understand that forgetting the math content knowledge one has previously studied is common for almost all students. Explore the topic of relearning—and that most people can relearn more rapidly than they did in their original learning.

Your students can introspect (do metacognition) on how much of the math they studied in the past they have forgotten. You and your students can explore ideas on what is remembered over time, what is forgotten because it was never understood and/or is not used, and how past learning helps in relearning. In addition, this is a good time to discuss study skills. Cramming for tests (memorize, regurgitate, and forget) is a common study skill! It does not lay a good foundation for relearning.

Another approach to this prerequisite for Math Methods content is through use of a mathography assignment. In such an assignment, students write about and reflect on their math backgrounds, experiences in and out of school, and attitudes. My 2/7/2016 Google search of the expression *mathography* produced over 5,000 results. This activity can be used with upper elementary school students and above.

Taking personal responsibility is one of the things to stress in these prerequisite activities. Every student needs to learn to take personal responsibility for meeting prerequisites in a course. This type of learning and responsibility-taking needs to be fostered, beginning at the earliest grades in school. Among other things, this means that every teacher needs to have knowledge and skill in helping students learn to self-assess and self-remediate. By the time students finish high school, they should be quite skilled in this aspect of their education.

These same types of issues apply to your students' knowledge and skills in the area of calculators and computers. Of course, you and your students know how to use a calculator. However, do you and they know how to use and how to explain the use of the M+, M-, MR, and MC keys on an inexpensive calculator? Do they know how to explain and deal with limited precision decimal arithmetic? How does the calculator's number line compare with the real number line? How does one learn to detect and correct calculator keyboarding errors? These types of questions are important to your students and to the students they will teach.

Next, consider computers and the more expensive calculators. Software now exists that can solve many of the types of problems students study in math courses up through the first two years of college. If a calculator or a computer can solve a particular type of math problem, what do we want students to be learning about solving that type of problem using by-hand methods? This is a difficult question. If you are not comfortable and convinced in your own personal responses to this question, how can you expect your students to learn to deal with it?

And, speaking of problem solving, what do your students really know about problem solving? Do they realize that problem solving is not only a key idea in math, it is also a key idea in every academic discipline? Do they understand how important math can be as an aid to helping solve the problems in non-math disciplines? Do they know how to integrate math problem solving throughout the non-math curriculum?

Here is an activity that you might want to use with your students near the beginning of the term. It is a short quiz that can be done by students working individually and then debriefing in small groups. It is not an activity to be turned in and graded. By listening to the small group discussions, and later through a whole class debriefing, you can gain some insights into the math backgrounds and understandings of your students.

1. What is a math problem, and what does the term "problem" mean in other disciplines?
2. Give an example of a math problem that has exactly one correct answer.
3. Give an example of a math problem that has exactly two correct answers.
4. Give an example of a math problem that has an infinite number of correct answers.
5. Give an example of a math problem that has no correct answer.

Part of the goal of this activity is to break students of the habit of talking about "finding **the** answer" to a math problem or to a problem in any other discipline.

Variations of this activity can be used at other times during the course. For example, what kinds of problems do historians (or name any other non-math discipline) try to solve? How do they make use of math in such endeavors? What aspects of problem solving that one learns by studying math are applicable to problem solving in other disciplines such as the social sciences, fine arts and performing arts, and so on?

Prerequisites for Math Methods Faculty

Prerequisites for higher education faculty members is certainly a delicate and sometimes controversial issue. What constitutes a good preparation to be a teacher of a Math Methods course? It seems to me it goes without saying that a Math Methods teacher should be a good teacher. I believe a good teacher of a Math Methods course needs to have math and math-education-related knowledge and skills that greatly exceed the prerequisites expected of students in the course. Moreover, the faculty member needs to know a great deal about elementary school students, their capabilities and limitations, how to help them learn, uses of math in everyday life and in both math and non-math disciplines of study, and so on.

Like reading and writing, arithmetic (more generally, math) is a part of every discipline that elementary school students study. Thus, each subject a teacher teaches is grist for the mill for illustrating uses of math. A Math Methods teacher needs to be able to draw examples from across the total curriculum their students are preparing to teach.

However, there is much more. Here are some of my ideas for you to think about. Self-assess yourself in the areas listed below. For areas in which you give yourself a relatively low rating, can you justify (in your own mind) your current level of preparation? (Perhaps you can provide good arguments that the topic is not relevant to being a god teacher of a Math Methods course.)

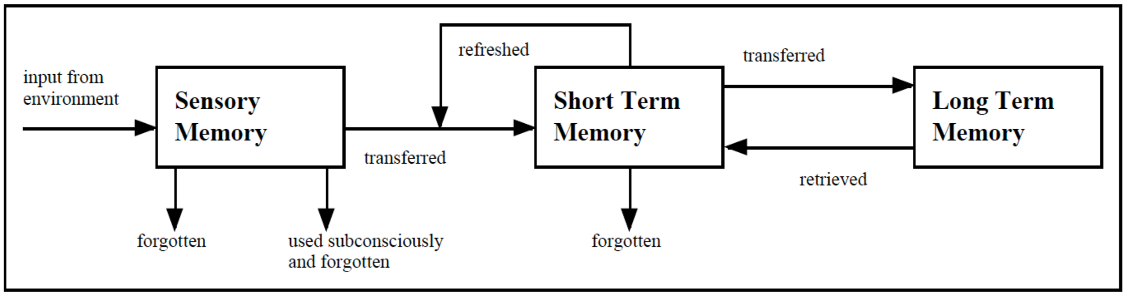
- Brain science.

    How do you detect if a student has dyslexia, dysgraphia, math anxiety, ADHD, or other brain disabilities that relate to learning and doing math? What are some other brain-based differences in students that are relevant to learning and doing math? See my book

    Progress in understanding how our brain works is helping us to understand better how to help students learn. See my book:

    Moursund, D. (2015). *Brain Science for Educators and Parents.* Eugene, OR: Information Age Education. Microsoft Word file at http://i-a-e.org/downloads/free-ebooks-by-dave-moursund/205-introduction-to-problem-solving-in-the-information-age-2/file.html and PDF file at http://i-a-e.org/downloads/free-ebooks-by-dave-moursund/206-introduction-to-problem-solving-in-the-information-age-3/file.html. Web: http://iae-pedia.org/Brain_Science.

Here is a brain memory diagram that I have found useful:



- Common Core State Standards.

  How are the Common Core State Standards changing teaching, learning, and assessment in K-12 math education? Online assessment is a part of this attempt to improve math education. These assessments have stirred up a hornet's nest (as of 2015 and early 2016). Are you familiar with such assessment instruments?

- Computational thinking.

  Computational thinking is taking into consideration the capabilities and limitations of calculators and computers as one is learning and using math to represent and solve problems. It includes computer modeling—a subject that is highly dependent on math modeling.

- Computer and Information Science (CIS).

  CIS is now a well-established academic discipline of study, research, and application. Computers are now an aid to representing and helping to solve the problems in every academic discipline. Ask yourself: Do you as a Math Methods teacher have sufficient knowledge and skills in CIS to prepare your students to adequately deal with the calculator and computer-related aspects of math in the elementary school curriculum? Nowadays, a number of computer games have some computer programming-like characteristics. What is your level of skill in computer programming?

- Computer-Assisted and Distance Learning and Online Learning.

  What constitutes "good" computer-assisted and distance learning materials? Are there substantial differences in various students' abilities to make effective use of these learning aids? What materials of this type are now widely used in elementary schools and in the homes of elementary school students?

- *Deeper Learning* research findings.

  Quoting from an American Institutes for Research article: http://www.air.org/resource/deeper-learning:

> [Deeper learning is] the combination of (1) a deeper understanding of core academic content, (2) the ability to apply that understanding to novel problems and situations, and (3) the development of a range of competencies, including people skills and self control.

- Fluency in speaking, reading, and writing in the language of mathematics.

  This can be a tricky issue. The teacher may need to role model communicating with elementary school students who are at various grade levels in terms of math maturity. In addition, Math Methods teachers need to communicate about math content and math methods at a level that is appropriate to their students and that also helps to raise the students' levels of math communication fluency.

- Learning theory.

  What learning theories are particularly relevant to learning and teaching math? What are good ways to teach for transfer of math learning to problem solving outside of the classroom and to problem solving in the various disciplines students study in school?

- Math maturity and math cognitive development.

  Math maturity increases through appropriate instruction and learning experiences, and also as one's brain matures. How can you tell if a student has achieved a level of math (cognitive) ability to learn and understand a particular math topic?

- Math tutoring

  Tutoring can be done by students, by unpaid volunteers, by parents, by paid staff, and so on. Paired learning or other forms of students working together can be considered as a type of tutoring. See the following free book:

  Moursund, D., & Albrecht, R. (9/2/2011). *Becoming a better math tutor.* Eugene, OR: Information Age Education. PDF: http://i-a-e.org/downloads/doc_download/208-becoming-a-better-math-tutor.htm. Microsoft Word: http://i-a-e.org/downloads/free-ebooks-by-bob-albrecht/220-becoming-a-better-math-tutor-2.html.

- Problem solving in math and non-math disciplines. See also Posing and answering questions.

  Problem solving lies at the heart of mathematics. How does one integrate problem solving into every math lesson? How does one assess a student's progress in getting better at problem solving? Can students learn to effectively self-assess in this aspect of learning math? See the *Deeper Learning* item earlier in this list.

- Special needs students.

  Logical/mathematical is one of the nine areas of intelligence that Howard Gardner discusses in his theory of multiple intelligences. How does one effectively teach math to a group of students having a wide variation in their math IQs?

- Talented and gifted students.

  Ask the same question as that given above. A typical elementary school class may have one or more students who learn math half as fast (and not as well, and with lower retention rates) as average students, and one or more who learn math twice as fast (and better, and with grater retention rates) as average students. Remember that this situation tends to continue for the students year after year.

- Uses of games in education.

  Games tend to be intrinsically motivating to students. What are some effective ways intellectually challenging games can be used in math education at the elementary school level?

It is easy to extend this list of possible faculty prerequisites. Perhaps now you understand why I believe that it is such a challenge to be a good teacher of a Math Methods course!

If your students are reading this book, you might want to engage them in a discussion about which of these topics they feel are relevant to being an elementary school teacher and their current qualifications in these various areas.

Chapter 4

Learning Theories in Math Education

Research in education has led to the development of many different learning theories. Thus, you are probably familiar with several such theories. Ask yourself: Do you know and make use of one or more learning theories that you feel are especially appropriate to the teaching and learning of math?

Here is an idea to try out with your Math Methods students. In a whole class or small group discussion mode, ask your students to name some of the learning theories they are familiar with. Then have them name one or two that they believe are especially important in math education at the elementary school level, and explain why these are especially important.

My personal response includes:

- Constructivism.
- Transfer of learning: High-road and Low-road transfer of learning.

There are, of course, many more learning theories. The next two sub-sections discuss the two I have listed.

Constructivism in Math Education

Constructivism is a learning theory that pinpoints a difficulty faced by all education systems. In brief summary, the theory says that students learn by building, or "constructing," upon their previous knowledge and understanding.

Catherine Fosnot is the director of Math in the City, a research and development project that provides teacher training in constructivist practice, aligned with the NCTM Standard. She is an international leader in constructivism in math education.

Constructivism is applicable in any discipline. It is especially important in math education and in other vertically structured curriculum areas. Quoting from the Catherine Fosnot site:

> **What is Constructivism?** "Students need to construct their own understanding of each mathematical concept, so that the primary role of teaching is not to lecture, explain, or otherwise attempt to 'transfer' mathematical knowledge, but to create situations for students that will foster their making the necessary mental constructions. A critical aspect of the approach is a decomposition of each mathematical concept into developmental steps following a Piagetian theory of knowledge based on observation of, and interviews with, students as they attempt to learn a concept."

Piaget, for example, was a strong supporter of constructivism. Quoting again from the Catherine Fosnot site:

> A central component of Piaget's developmental theory of learning and thinking is that both involve the participation of the learner. Knowledge is not merely transmitted verbally but must be constructed and reconstructed by the learner. Piaget asserted that for a child to know and construct knowledge of the world, the child must act on objects and it

> is this action which provides knowledge of those objects (Sigel, 1977); the mind organizes reality and acts upon it. The learner must be active; he is not a vessel to be filled with facts. Piaget's approach to learning is a readiness approach. Readiness approaches in developmental psychology emphasize that children cannot learn something until maturation gives them certain prerequisites. The ability to learn any cognitive content is always related to their stage of intellectual development. Children who are at a certain stage cannot be taught the concepts of a higher stage.

Now, think about what a preservice elementary teacher might have learned about constructivism before beginning to take teacher education courses. The whole idea of prerequisites is based on constructivism. Thus, preservice teachers have repeatedly experienced lessons in which their teacher begins by reviewing some of the prerequisite knowledge assumed in a lesson.

This is a constructivist approach—but most teachers do not make it explicit that they are using a constructivist approach. Moreover, math is a vertically structured discipline of study. Thus, any new material may well draw upon (be built on) content that a student has supposedly learned in previous math instruction in the past weeks, months, and years. However, it is not feasible to begin each new math lesson with a review of all previous math lessons.

This may well be the root of the "I can't do math" beliefs of many adults. In a vertically-sequenced course of study, it is quite easy for a student to encounter new material that assumes prerequisite knowledge and skills that were previously covered, but that were not understood and/or learned very well by that student, or that have been forgotten.

As a Math Methods teacher, you probably will want to role model constructivist math teaching and learning, both to help your students with the current lesson and to help them to gain an increased level of expertise in constructivism.

High-road and Low-road Transfer of Learning

Teaching for transfer is one of the seldom-specified but most important goals in education. We want students to gain knowledge and skills that they can use both in school and outside of school, both immediately and in the future.

An excellent introduction to this topic is given in an article by David Perkins and Gavriel Solomon, Transfer of Learning.

> Perkins, D.N., & Salomon, G. (9/2/1992). Transfer of learning. Retrieved 2/21/2016 from http://webcache.googleusercontent.com/search?q=cache:S5YRMwaCkXoJ:jaymctighe.com/wordpress/wp-content/uploads/2011/04/Transfer-of-Learning-Perkins-and-Salomon.pdf+&cd=1&hl=en&ct=clnk&gl=us).

My free book, *Learning Problem-solving Strategies Through the Use of Games: A Guide for Teachers and Parents* (January, 2016), strongly emphasizes and illustrates teaching for transfer of learning.

> Moursund, D. (January, 2016). *Learning Problem-solving Strategies Through the Use of Games: A Guide for Teachers and Parents.* Eugene, OR: Information Age Education. PDF file: http://i-a-e.org/downloads/free-ebooks-by-dave-moursund/279-learning-problem-solving-strategies-through-the-use-of-games-a-guide-for-teachers-and-parents-1.html. Download the Microsoft Word file: http://i-a-e.org/downloads/free-ebooks-by-

dave-moursund/278-learning-problem-solving-strategies-through-the-use-of-games-a-guide-for-teachers-and-parents.html.

Quoting from my book:

> The low-road/high-road theory of learning has proven quite useful in designing curriculum and instruction. In low-road transfer of learning, one learns a task to automaticity, somewhat in a stimulus/response manner. When a particular stimulus (a particular situation) is presented, the prior learning is evoked and used. The human brain is very good at this type of learning.
>
> **Low-road transfer** of learning is associated with a particular narrow situation, environment, or pattern. The human brain functions by recognizing patterns and then acting upon these patterns. Consider the situation of students learning the single digit multiplication facts. This might be done via work sheets, flash cards, computer drill and practice, a game or competition, and so on. For most students, one-trial learning does not occur. Rather, a lot of drill and practice over an extended period, along with subsequent frequent use of the memorized facts, is necessary.
>
> Many students find that they have difficulty transferring their arithmetic fact knowledge and skills from the learning environment to the "using" environment. One of the difficulties is recognizing when to make use of the memorized number facts. In school, the computational tasks are clearly stated; outside of school, this is often not the case.
>
> This helps to explain why rote memory is often useful in solving routine, frequently occurring problems, but critical thinking and understanding are essential in dealing with novel and challenging problems. It also supports the need for broad-based practice even in low-road transfer. We want students to recognize a wide range of situations in which some particular low-road transfer knowledge and skills is applicable.
>
> Math education in schools tries to achieve an appropriate balance between rote memory and critical thinking by making extensive use of word problems or story problems. In word problems, the computations to be performed are hidden within a written description of a particular situation. The hope is that if a student becomes better at reading and deciphering word problems—extracting the computations to be performed and the meaning of the results—this will transfer to non-school problem-solving situations and solving math problems in other disciplines.
>
> It turns out that it is quite difficult to learn to read well within the discipline of mathematics. Many students have major difficulties with word problems and with learning math by reading math textbooks. Their depth of understanding of math and their ability to read math for understanding stand in the way of their being able to deal with novel, challenging math problems they encounter.
>
> **High-road transfer of learning** for improving problem solving is based on learning some general-purpose strategies and learning how to apply these strategies in a reflective manner. The *build on previous work strategy* is an excellent candidate to use to begin (or, expand) your repertoire of high-road transferable problem-solving strategies. To do this, think of a number of personal examples in which you have used this strategy as an aid to problem solving. Mentally review what you did in each case. In the near future, each time you make use of this strategy, consciously think about its name and the fact that you are

using it. Also, in the future when you encounter a challenging problem, consciously think through your repertoire of high-road transferable problem-solving strategies. Your goal is to increase your ability to draw upon this repertoire of aids to use when faced by a challenging problem.

The *break a problem into smaller problems strategy* is another example of a high-road transferable strategy. This strategy is often called the *divide and conquer strategy,* and that is the name that will be used in the remainder of this book. It is helpful to have short, catchy names for strategies. A large and complex problem can often be broken into a number of smaller, more tractable problems. It is likely that many of your students do not have a name for the strategy and do not automatically contemplate its use when stumped by a challenging problem.

Chapter 5

A Teacher's Collection of Professional Resources

Over the years of preparing to become a teacher and then being a teacher, every teacher builds a personal professional collection of teaching resources. Many of these resource materials are stored only in the teacher's brain. However, quite a bit can also be stored in physical filing cabinets, on shelves, and electronically in computers. Much of the content of this chapter is drawn from the IAE-pedia article, Math education digital filing cabinet.

Personal Knowledge and Skills

A teacher's personal mind/body knowledge and skills are used for **just in time decision making and implementing the decisions.** Talking to a class, responding to a question, observing and facilitating a group of students engaged in a team activity, and writing on a chalkboard/whiteboard/Smartboard all require making use of one's mental and physical knowledge and skills. In essence, these are all rapid response situations.

Reading and writing allow us to supplement the knowledge and skills we store in our heads. Skill in storing, retrieving, understanding, and making use of retrieved information is now one of the basics of education. All students need to develop their own personal balance between what they store in their heads, store on paper, and store in computerized information storage and retrieval systems.

Mental and physical knowledge and skills improve through practice (including studying) and use. Both decline from one's peak performance level through disuse. This decline from peak performance is a key concept in teaching and learning. Most students forget most of what is covered in a course unless they are in a situation where it is is used and the knowledge and skills are periodically refreshed.

So, as a teacher of teachers, you should think about this very carefully. Suppose that your students will forget well over three-fourths of what you cover in your class by the time a year has gone by. How does this knowledge affect what and how you teach preservice math teachers?

- One of the things that we know is that relearning can be quite a bit faster than initial learning. What do you do in your teaching to help students prepare for the relearning tasks they will encounter throughout their professional careers?
- What parts of the content do you really, really want your students to remember for a very long time? Is it the same for each student? Note how this ties in with constructivism and with individual differences. This is a challenging question.
- When your students become teachers and begin to teach math, what do you want them to do for their students in terms of the two questions given above?

An elementary school teacher faces the challenges and responsibilities discussed above over the full range of courses he or she is teaching. The Math Methods teacher needs to focus special attention on the math-related aspects of these challenges and responsibilities.

Physical Materials: Physical Filing Cabinets

Think of some of the physical things that an elementary school math teacher is apt to want to have readily available. These might include:

1. A personal library of math-related books, journals, magazines, and articles.
2. Math-related books and pamphlets for student use.
3. Math-related teaching supplies for use by the teacher and students. This might include a classroom set of rulers, protractors, compasses, calculators, math manipulatives, colored pencils, colored chalk or white board markers, colored paper, graph paper, geoboards, and so on.
4. Equipment to project material on a screen. For a great many years, this was an overhead projector with acetate "slides" and erasable markers. An overhead projector is still a valuable aid. However, document cameras and a computer system with projector are now more widely used in the industrial developed nations.
5. Grade books, samples of work done by students in the past, and perhaps a student-created plus teacher-created math portfolio for each student.
6. CDs and DVDs (and perhaps also videotapes) used in teaching math.
7. A personal computer and/or tablet computer.
8. A Smartphone.
9. Etc.

Some of these materials will be supplied by the school, and some will be in a teacher's private collection, often purchased with the teacher's own money.

> Here is an aside. What happens when a teacher moves to teaching at a different grade level or moves to another school or district? It would surely help if all of one's own personal materials were carefully labeled and/or on an inventory list. It would also help a preservice teacher to know what materials are apt to be (should be) supplied by the school.

If the total collection of physical resources is relatively small and is used frequently, then it may well be possible to remember where everything is, when it will be needed, how it will be used, and so on. The retrieval process might be completely dependent on one's own mental storage and retrieval capabilities.

However, as the collection grows, many teachers find this organization and retrieval task to be mentally overwhelming. The use of physical filing cabinets to deal with the paper parts of such a collection goes back more than a hundred years. With a little instruction or through trial and error, teachers learn how to organize paper materials into file folders and store the folders in alphabetical or subject matter order in a filing cabinet. While this may seem like a simple task, it isn't. Where do you file a document that might well fit into several different folders? How do you find such a document years later?

Virtual Materials: Personal Digital Filing Cabinets (PDFC)

I have incorporated the idea of a personal digital filing cabinet (PDFC) into a number of courses that I have taught. I have found it useful for every student in my classes to have a PDFC, to add to it during the course, and to develop/find materials that others in the class may want to add to their own PDFCs.

A student's PDFC might be stored as word processed documents, spreadsheets, and other files on the student's personal computer. Alternatively or in addition, parts or all of it might be stored on the Web.

My PDFC consists of two parts. First, I have a specific website where I store math materials I want to share with others and refer back to from time to time. As of 2/3/2016 this file has had nearly 60,000 hits—mostly from people other than myself.

Second, I consider the entire IAE collection of materials to be a personal digital filing cabinet that I share with the world. As of 2/18/2016 the IAE materials have received about 10 million hits. I make use of these materials many times a week, and I spend much of my spare time updating and adding to this collection. Over the years, I have developed a large collection of Math Education Quotes as well as a collection of more general educational quotes, Quotations Collected by David Moursund. I make frequent use of these in my writing, and the math quotes have proven to be a very popular page in the *IAE-pedia.*

The key idea in a PDFC is that the person who creates and maintains the file has invested the time to thoroughly peruse and understand its contents. Thus, for example, I recently did a Google search of the expression *free math education videos.* I got over 90 million results. I can file away in my brain that there are likely a very large number of free math education videos, and I might add to my PDFC this fact and a few key Web addresses. But, what I really need to do is to go to some of these sites and view a number of videos. I need sufficient personal knowledge of the video links that I put into my PDFC so I know each is suitable for use in my teaching. After I have used it with my students, I will add some comments about its effectiveness to my PDFC.

The Web

> Knowledge is of two kinds. We know a subject ourselves, or we know where we can find information upon it. (Samuel Johnson; English essayist, literary critic, biographer, editor, and lexicographer; 1709-1784.)

Here is a comparison I find useful. A Personal Digital Filing Cabinet (PDFC) is to the Web as the books in one's personal library are to a gigantic public library. I have personal ownership of my PDFC and of my books. I am familiar with their contents. They are like an auxiliary memory for me.

The Web is not only the world's largest library, it also provides access to software that can help to solve a wide variety of problems and accomplish a wide variety of tasks.

Every person teaching math is faced by the issue of what students should learn to do mentally, what they should learn to do with "simple" aids such as pencil, paper, ruler, and other "by hand" tools, and what they should learn to do with more powerful aids such as calculators, computers, and the Web.

People who use math in their everyday lives and occupations make personal decisions based on their knowledge and skills, availability of aids, requirements of a job, and so on. Eventually they develop "ownership" of their personal ways of doing and using math.

As you know, math has both great breadth and great depth. One aspect of learning math is learning to build on the accumulated available mathematical knowledge. A modern education prepares students to gain the knowledge and skills needed to make use of the readily available accumulated knowledge of the human race.

Thus, I believe that in each area that students study, they should be learning to make use of the accumulated knowledge that is available to them. I believe that you, as a Math Methods teacher (or, as a student in such a course) need to openly address this issue. This means, for example, that Math Methods students should receive explicit instruction in how to search for, retrieve, read and understand, and use "math stuff" available on the Web.

Materials That Add Joy to Learning

Advice to preservice and inservice elementary teachers. Build a personal collection of materials that you can use to add more joy to the lives of your students. Jokes, personal stories, toys you bring to class, games and other materials you use in teaching—all can be useful. Information Age Education is doing a sequence of newsletters on the Joy of Learning. The series began with Issue # 175, December, 2015. In the second of this newsletter series, I write about Joy in Learning and Playing Games. My colleague Bob Albrecht has spent much of his professional career developing games for use in math education and a number of his games are available online (free).

Chapter 6

Sustainability as a Math Education Topic

You may think that this short chapter is somewhat "off the wall" and outside the realm of preparing K-8 teachers of mathematics. One of the challenges in math education is to find problems that are important and relevant to students and that make use of the math they are learning. So, I am always on the lookout for uses of math in the various non-math disciplines that interest me. Sustainability is one of these topics.

A great many people believe that sustainability is a major problem facing the people of our world. Quoting from the Wikipedia:

> One of the first and most oft-cited definitions of sustainability, and almost certainly the one that will survive for posterity, is the one created by the Brundtland Commission, led by the former Norwegian Prime Minister Gro Harlem Brundtland. **The Commission defined sustainable development as development that "meets the needs of the present without compromising the ability of future generations to meet their own needs."** The Brundtland definition thus implicitly argues for the rights of future generations to raw materials and vital ecosystem services to be taken into account in decision making. [Bold added for emphasis.]

It is possible to incorporate the sustainability idea into the content of many different courses. For example, language arts students might be given assignments that include reading and writing about sustainability. In science they might study the effects that dams have on efforts to sustain fish populations.

Many math textbooks used at the precollege level include examples that can be construed as being related to sustainability. For example, here is a word problem:

> Mary wants to buy a cashmere sweater that costs $62. She has an allowance of $3.50 a week. She figures that she can save $2 per week from this allowance. How many weeks will it take her to save enough money to buy the sweater?

Notice that the cashmere sweater problem is a consumption-oriented problem situation. Mary has income and she "wants" to buy a cashmere sweater. There is no indication of why she wants the sweater or if she "needs" the sweater. What is cashmere? Are there other types of sweater fabric that could be more sustainable? How does this tie in with Mary's carbon footprint?

Aha! At what age can students begin to understand some of the basic ideas of sustainability, carbon footprint, ways to save energy, and so on? Are there math problems that are appropriate to the math curriculum for young students that can help to teach and support sustainability?

I have two goals here:

1. Help preservice and inservice elementary school teachers of math realize that they can and should be teaching students about math-related aspects of sustainability.
2. Provide some guides to resources that will help in teachers implementing such ideas.

A number of websites aid users in calculating carbon footprints.

The cashmere sweater word problem relates to a category of problems that I consider to be divorced from reality. Ask yourself: Is this word problem realistic for most students? What if you are a student living in poverty? What "values" are taught using this problem?

Sustainability and quality of life are related. And, as might be expected, the availability of a good education is an important component of quality of life. I have written several articles about quality of life and measures of quality of life. I believe that this is a suitable topic for upper elementary school students. It can provide good practice in thinking about how to quantify and measure various components that students feel contribute to their quality of life. (Hmm. I wonder how your students might measure/rate their precollege education, their college education, or their math education at those two levels.)

I enjoy collecting Math Word Problems Divorced from Reality. Some word problems give me quite a laugh. It is difficult to create word problems that are relevant to students' lives, and that provide good examples of the use of the math that students are studying.

Chapter 7

Supportive Professional Organizations

In my home state (Oregon) we have a professional organization Teachers of Teachers of Mathematics (TOTOM). It has annual meetings and a website. Quoting from the TOTOM website:

> TOTOM is dedicated to providing a forum for people involved in the training of mathematics teachers. Each year during September TOTOM organizes a meeting for the sharing of information on teacher licensure and training. This includes both elementary, middle, and high school teacher training. Members are typically members of the mathematics and education departments of the community colleges, universities and colleges in Oregon.

It is my observation that many teachers of Math Methods courses and Mathematics for Elementary Teachers courses work in relative isolation. Many teacher education programs have only one or two faculty members who teach the Math Methods course for preservice elementary school teachers, and many Math Departments have only a few faculty who teach the Mathematics for Elementary Teachers course.

Not infrequently, one of both of these courses is taught by adjunct faculty members who are only on campus when they are teaching their course and holding their office hours. Others may be teaching the courses completely online. Thus, many of these teachers of preservice math teachers have relatively little regular opportunity for close and continuing professional interaction with others in their field.

My suggestion is that the profession of being a teacher of courses for preservice and inservice math teachers can be improved by a greater emphasis on building local and regional communities of practice.

And, of course, there are state and national structures to help such faculty. Most states have a state math organization that is affiliated with the National Council of Teachers of Mathematics. The National Council of Supervisors of Mathematics is a valuable player in Math Education. The Association of Mathematics Teacher Education provides links to 18 professional societies.

Chapter 8

Final Remarks

I consider this book to be a **Work in Progress**. My continuing goal in this writing project is to stimulate my thinking and my readers' thinking. The Web makes it easy for me to revise and add to the materials that I make available free through Information Age Education. Perhaps you will decide to draw ideas from my materials as you develop and write your own ideas. As you do this, I hope you will share them with me and with others.

The rate of progress in technology is both fast and accelerating. Some of this progress is now affecting and will further affect the processes we use in teaching, assessment, and learning. And, of course, some of this progress will affect the content that we teach at various grade levels. Thus, being a good teacher will require a career-long learning effort on the part of both elementary school teachers and teachers of teachers.

If you want to stretch your mind about some of the possibilities that may be coming down the pike, see the *IAE Blog* entry, Education for the Coming Technological Singularity. The *singularity* is when computers become *smarter* than humans. In some ways, they already are!

I wish the best of success to all of you in you future math education endeavors.