Syllabus

Increasing the Math Maturity of K-8 Students and Their Teachers

Some small corrections were made 5/27/2011, 6/5/2011, and 9/26/2013

This is a detailed syllabus for a 10-week (three quarter hours or two semester hours) graduate course developed by David Moursund. The course was offered through the University of Oregon Continuation Center as a distance education course Spring term, 2010.

The course is designed for inservice elementary school teachers with an interest in improving their knowledge and skills in teaching math. The materials are suitable for self-instruction.

This detailed syllabus and a variety of materials available free on the Web constituted all of the required reading for the course. The links have been checked and work correctly as of 5/21/2011.

David Moursund

(Learn more about David Moursund at http://iae-pedia.org/David_Moursund.)

"In a completely rational society, the best of us would be teachers and the rest of us would have to settle for something less, because passing civilization along from one generation to the next ought to be the highest honor and the highest responsibility anyone could have." (Lee Iacocca, American industrialist; 1924-.)

"Mankind owes to the child the best it has to give." (United Nations Declaration of the Rights of the Child, 1959)

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Housekeeping and Other Details

> "Education is a human right with immense power to transform. On its foundation rest the cornerstones of freedom, democracy and sustainable human development." (Kofi Annan; Ghanaian diplomat, seventh secretary-general of the United Nations, winner of 2001 Nobel Peace Prize; 1938-.)

The section provides some "housekeeping" details such as rules, regulations, expected student performance, and so on.

Brief Course Description

A course for teachers of math and science focusing on increasing K-8 student levels of math maturity; includes an emphasis on computational thinking and problem solving across the curriculum. Course participants must have Email and Web access as well as access to elementary or middle school students in order to carry out a number of the course assignments.

Time and Quality Requirements

The total expectation is 120 hours of effort. The course syllabus is based on dividing the 120 hours of the course into 10 Units (Lessons) of about 12 hours each.

This is a graduate course. The expectations and standards will be similar to those in other graduate-level courses in high quality colleges and universities. Participants are expected to have the maturity and interest to take personal responsibility for their own learning and to do high quality work in the assignments.

This course is specifically designed for inservice teachers. It may also fit the needs of others, such as people who have completed a teacher certification program but are not yet employed as a teacher or people who were teachers in the past and are no longer teaching. In any case, to complete the required assignments in the course participants will need to have access to a classroom of students.

Plagiarism and Other Types of Student Academic Misconduct

Plagiarism and other types of student academic misconduct will not be tolerated. Various course assignments require developing and implementing lesson plans and a variety of written activities. It is expected that you will do your own work and that you will adequately cite the various reference materials you draw on.

This is a University of Oregon course. The University of Oregon has a Student Conduct Code. For details see http://studentlife.uoregon.edu/StudentConductandCommunityStandards/StudentConductCode/tabid/69/Default.aspx.

Assignments and Grading

The course has 10 graded assignments of 100 points each. This makes a total of 1,000 possible points. There are no opportunities or options for extra credit. Some of the assignments will take more time and effort than others. Some course participants will find that the course takes less time than the 120-hour estimate, and some will find that it takes more time.

The typical lesson-grading situation is that a lesson will be submitted electronically, graded along with some feedback comments inserted into the document, and returned electronically. Occasionally a grade of “Redo” will be assigned. This typically means that the submitted assignment has major flaws due to a significant misinterpretation of the assignment. (Sometimes this misinterpretation indicates a poorly stated assignment and will lead to an improvement in the syllabus!)

This in a graduate course. The grading scheme is:

920 to 1,000 points = A minus to A plus.

840 to 919 points = B minus to B plus.

760 to 839 points = C minus to C plus.

759 or less points = NP (No Pass; Failure).

If a student is enrolled under a Pass/No Pass option, a grade of Pass is equivalent to a B minus or higher.

Each assignment is designed to give you the opportunity to show that you have done and understood the reading and video viewing assignment and that you have made progress in integrating this information with your current knowledge and skills. Many of the assignments require you analyze what is going on in your classroom and to try out some new ideas in your classroom. Such assignments are relatively open ended.

Length of Time in Which to Complete the Course

This course is designed to be use in both a quarter system and a semester system of higher education. In a quarter system, a course is typically 10 weeks in length and is followed by a week during which final exams are given. In a semester system a semester is usually 15 weeks in length and is followed by a week during which final exams are given.

If there are a “critical mass” of enrollees, a course email-based Discussion Group List will be established. All participants in the course will be able to post to this list, and each posting to the list is automatically emailed to all members of the list. Only the course instructor or instructors and the students can post to the list. Of course, both the course instructor(s) and the course participants can email individually to each other. Users of the list are requested to use common sense and care to avoid sending messages to the entire list when their intent is to send a message to just one person or a small group of people on the list.

Course participants can move through the course at their own pace, subject to the following:

1. Please do not do more than two Units (Lessons) a week. Under ordinary circumstances, the course instructor will grade submitted assignments within three days of their receipt. Under ordinary circumstances students are expected to take into consideration the feedback they receive on a Unit before doing and submitting the next Unit.
2. Students are expected to complete the coursework during the term in which they have enrolled. If special circumstances acceptable to the course instructor make it impossible for a student to follow this schedule, the instructor has the options of awarding an Incomplete. A student requesting an Incomplete must provide an explanation and justification for the request, and a detailed timeline

for when the work will be completed. The course instructor makes the decision as to whether the grade of Incomplete will be awarded and whether the proposed timeline is acceptable.

Unit # 1: Introduction and Getting Started

> "The longest journey begins with the first step." (Ancient Chinese proverb.)
>
> "Learning without thinking is labor lost; thinking without learning is dangerous." (Chinese Proverb)

This is the first of 10 Units (Lessons) in the course. It is designed to get you off to a successful start, and it covers a miscellaneous variety of topics related to the course.

The goal in this course is to help you get better at helping your students get a better math (or, math and science) education. Thus, much of the content and a number of the assignments have the flavor of translating theory into practice. The course will give you increased insight into a number of research-based (theory-based) ideas. The course is successful to the extent that you incorporate such ideas into your teaching in a manner that leads to your students getting a better education.

Distance Education and Some Brain Theory

The Syllabus materials you are currently reading are one of the components of a Distance Education course. The course materials, assignments, and feedback from a course instructor are designed to help you learn.

All of these learning aids are external to (at a distance from) your brain and the rest of your body. None of the materials are connected directly into your neurons by a hard-wired or wireless connection.

All learning takes place within one's brain and the rest of one's body. The information to be learned can come from: 1) sources internal to your body (including from your brain); and 2) sources outside your body via your senses. A books, video, and email are external source of information.

As an example of (1) above, suppose that I am in a sensory deprivation tank. While in the tank, I can still think about information stored in my brain, I can learn by combining this information in new ways, and I can pose and solve problems. For another example, progress in cognitive neuroscience (brain science) is helping us to better understand how learning and unlearning occurs at a subconscious level while one is asleep.

In terms of learning through one's senses, you have probably heard the saying: "You can lead a horse to water but you can't make it drink." Your senses receive lots of data. Much of this data is filtered out (ignored) before it makes its way into your brain. For an example of this, undoubtedly you have experienced the situation of your mind wandering from a topic a teacher is presenting. In essence, your brain is turning off data coming from the teacher into your eyes and ears.

Once data comes into your brain, much of it is processed at a subconscious level, and some is processed at a conscious level. Your brain is quite good at ignoring (not paying attention to) data that it decides is irrelevant or is less relevant than other data that is available for processing. One of the goals in schooling is to help you get better at focusing your attention on the content to be learned. One of the characteristics of a good teacher is being able to attract and hold students' attention. Attention has been a major and successful area of cognitive neuroscience research. See http://en.wikipedia.org/wiki/Attention.

The following somewhat simplified model of learning academic content consists of three key ideas:

1. All of the learning you do occurs inside your brain and the rest of your body. The learning process is actually occurring at a completely subconscious level—learning involves changes at the cellular level in your brain and in the rest of your body. New neural connections are crated, and existing neural connections are strengthened or weakened.
2. The data that is processed in a manner that leads to learning can come from internal and external sources. In both cases, the learning that occurs is based on (constructed on) what has been learned in the past.
3. Paying conscious, alert attention to the topics and ideas you are trying to learn, reflecting on them, and doing metacognition on them can help direct the subconscious learning process. (Note, however, the concepts in W. Timothy Gallwey's *Inner Game of Tennis.* In kinesthetic learning of sports, dancing, and so on, conscious attention and careful thinking can often get in the way of developing the mind/body subconscious interactive coordination and automaticity that is needed.)

Likely you think that this is three-item is a weird way to think about learning. In essence, it says that all schooling is a form of distance education. Reading a book is a form of distance education. A well-written book will tend to grab and hold your attention. Viewing a video, listening to an audiotape, listening to a faculty member present a lecture, and participating in a small group discussion are all forms of distance education.

However, these varying forms of education have varying levels of success in grabbing and holding a learner's attention, and they provide varying forms and levels of interaction and feedback. Research indicates that one-on-one tutoring by a skilled and knowledgeable tutor is a much more effective aid to learning than is participation in a typical large class environment. Here we use the term one-on-one tutor to include both the human tutor and other learning aids such as books, audio and video materials, field trips, and computerized tutoring systems. Such a tutor can, of course, have the tutee making use of books, manipulatives and laboratory equipment, audio and video materials, and so on. The one-on-one tutoring provides better individualization and better interaction (feedback) than does the large class environment.

At the current time, one-on-one highly individualized tutoring by a well-qualified human tutor is considered to be the "gold standard" in education. For the most part, our current "factory mode" versions of schooling do not come close to meeting the gold standard.

Our precollege and higher education systems are beginning to make considerable use of Information and Communication Technology (Internet, Web, audio and video materials, computers and computerized tools, etc.) as an aid to earning. In a few teaching/learning situations, we now have Highly Interactive Intelligence Computer-Assisted Learning systems that meet (and, I some special cases, exceed) the gold standard. Such materials are quite expensive to develop.

The course materials you are currently using certainly do not come close to meeting the gold standard. However, research indicates that a distance education course such as the one you are currently taking is, on average, as effective an aid to learning as a traditional college or university face-to-face course.

You can learn more about Distance Education at:

Staff development via distance education. Retrieved 5/21/2011 from http://iae-pedia.org/Staff_Development_via_Distance_Education.

This document includes a discussion of Highly Interactive Intelligent Computer-Assisted Learning (HIICAL). The cost of developing a HIICAL is perhaps 100 to 200 times the cost of developing the course you are currently taking. However, research indicates that the relatively inexpensive-to-develop courses are effective aids to learning. The learning outcomes from a course such as this are comparable to the learning outcomes from a traditional face-to face course. Research supporting this assertion is available in the meta-study:

Means, Toyama, Murphy, Bakia, and Jones (2009; revised 2010). Evaluation of evidence-based practices in online learning: A meta-analysis and review of online learning studies. U.S. Department of Education. Retrieved 5/20/2011: from http://www2.ed.gov/rschstat/eval/tech/evidence-based-practices/finalreport.pdf. (94 page report.)

Please note that there is no requirement that you access and read the two documents referenced above. Through taking the course, you will learn (mainly by experience) some things about online education. If you want to learn still more about online education, the two references given above may prove useful to you.

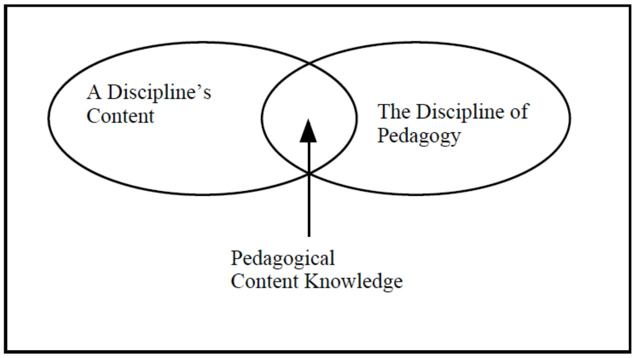
Quick Overview of the Course

The discipline of mathematics is divided into Pure Math, Applied Math, and Computational Math. The various disciplines in science are divided into Theoretical, Experimental, and Computational (with the terms varying somewhat among the different specific sciences). Here, "computational" refers to using brainpower and computer power to develop models (representations) of problems and to solve the problems.

Here is a very brief overview of what one does in studying and learning a discipline:

1. Learn some of the accumulated content knowledge of the discipline. The accumulated knowledge is often thought of in terms of data, information, knowledge, wisdom, and foresight. (This is a little confusing, since "knowledge" is used in talking about the totality, and knowledge is also one of the categories on the scale.)
2. Learn to think and solve problems using the accumulated knowledge, tools, and other aids to representing and solving problems in this and other disciplines. For example, reading and writing are tools that cut across all disciplines.
3. Learn about some of the history, culture, beauty, ways of teaching, and ways of learning the discipline

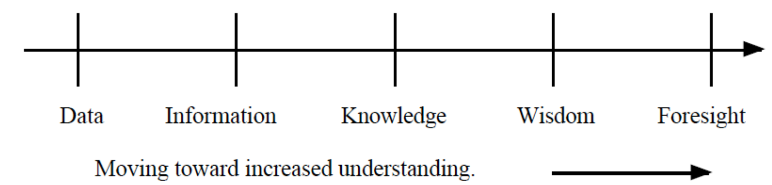
As a teacher, you are probably familiar with the ideas of content knowledge, pedagogical content knowledge, and pedagogical knowledge. See the Venn diagram given below.



Pedagogical content knowledge.

The course presented in this syllabus is a Pedagogical Content Knowledge course designed primarily for people who teach or help teach math at the K-8 levels. Thus, the main focus is on Items 2 and 3 in the 3-item list given above. Since many of its ideas carry over to science and since all teachers of science have a significant math background, the course is also quite suitable for teachers of science at the K-8 levels.

In Item 1 from the list given above, the word *knowledge* is used in a relatively loose fashion. The diagram given below suggests that knowledge can be thought about on a scale, and that learning in a content area such as math or science can be a balance among the five listed items.



Data: Raw, unprocessed facts and/or figures, often obtained via use of measurement instruments.
Information: Data that has been processed and structured, adding context and increased meaning.
Knowledge: Ability to use information tactically and strategically to achieve specified objectives.
Wisdom: Ability to select objectives that are consistent with and supportive of a general set of values, such as human values.
Foresight: Ability to accurately predict outcomes of one's proposed decisions and actions.

Short definitions of data, information, knowledge, etc.

It is assumed that you understand the educational theory called constructivism—that a brain learns by building upon and integrating with its current knowledge and skills. (See http://iae-pedia.org/Math_Methods_for_Preservice_Elementary_Teachers#Constructivism_in_Math_Education.) Each individual learner has a unique knowledge and skill set. Each learner thus faces a unique problem or task of learning. A teacher and other aids to learning are facilitators—but it is the learner that does the learning.

There are two unifying content themes topics in this course.

- Math Maturity (The basic ideas also apply to science maturity. Maturity in a discipline tends to endure over time even as one forgets some of the details and experiences a decrease in some of one's skills through disuse.)
- Problem Solving. (This includes an emphasis on word problems and relates closely to Computational Thinking. Problems in all disciplines are commonly represented using spoken and written language, gestures, sounds, and other methods of communicating with/through the learner's senses.)

Reading and Video-Viewing Assignment for Unit #1

1. Read the Course Syllabus that precedes this point in the document. Pay careful attention to the Unit #1 ideas presented above. Do a quick browse or scan of the remaining material so that you have some idea of where the course is going. Notice the References and Resources section at the end of the Syllabus. Many of the references are drawn from Dave Moursund's personal Electronic Digital Filing Cabinet. That is, they are part of the resources that he frequently draws on and makes available for sharing with students and colleagues.
2. Here are other Reading and Video-Viewing Assignments for Unit # 1. As you read and/or view these materials, think about your knowledge and skill in learning by reading and learning by viewing. You have spent years developing this knowledge and skills. What are your students' current abilities to learn math and science by reading and by viewing videos? What specifically are you doing to help them get better at this?
    - Apple (n.d.). Introduction to iPhoto (an ad). Retrieved 5/20/2011 from http://www.apple.com/welcomescreen/ilife09/iphoto/play/. (5 minute video. This video is an ad.). Think about how computers, math, and science combine ion helping people solve some problems related to the use of digital photography. Think about how the ready availability of such powerful computer tools may be affecting what we want students to learn in math and science in school.
    - Pinker, Steven (2008). The computational theory of mind, part 1 (11 minute video). Retrieved 5/20/2011 from http://www.youtube.com/watch?v=LVrb5ClvDho. The same link provides access to a number of related talks by Steven Pinker and other people. (See the menu on the right side of the Steven Pinker video page for parts 2 and 3 of his talk.)
    - Moursund, D.G. (2009). Two brains are better than one. Retrieved 5/20/2011 from http://iae-pedia.org/Two_Brains_Are_Better_Than_One. You and your students live in a world in which human brains, computer brains, and computerized tools all work together to solve problems and accomplish tasks. Think about your own insights into the capabilities and limitations of human and computer brains. Think about what you are doing in your teaching to help your students gain appropriate insights into the capabilities and limitations of human and computer brains— working individually and in cooperation—to solve the problems and accomplish the tasks that are covered in the curriculum you teach.

- Moursund, D.G. (2008). Computational thinking. Retrieved 5/20/2011from http://iae-pedia.org/Computational_Thinking. The key idea here is human and computer brains working together to represent (develop mathematical and computational models) and solve problems. Think about your own insights into math modeling and computer modeling, and the roles they play in problem solving across the curriculum. Then think about what you are doing to help both you and your students gain further insight into this two-brain approach to solving problems and accomplishing tasks.
- Moursund, David (2008). Introduction to using games in education: A guide for teachers and parents. Eugene, OR: Information Age Education. Access at http://i-a-e.org/downloads/doc_download/19-introduction-to-using-games-in-education-a-guide-for-teachers-and-parents.html. Read the two-page section on Modeling and Simulation that begins on page 20 of the book.
- Moursund, D.G. (2007). Introduction to problem solving in the Information Age. Eugene, OR: Information Age Education. Retrieved from http://i-a-e.org/downloads/doc_download/7-introduction-to-problem-solving-in-the-information-age.html. Read Chapter 9. This chapter provides some information about the math and computer modeling aspects of computational thinking.

Graded Assignment for Unit # 1

Assignment # 1 (100 points). All assignments are to be submitted as attachments to an email message sent to the course instructor. The assignment should be done using Microsoft Word or some other commonly used word processing system that includes a spelling check and sufficient desktop publishing capabilities to do an acceptable level of desktop publication of the document. It is assumed that submitted documents will have been carefully spell checked and be of a writing quality that one expects of graduate students.

Unless otherwise specified, there are no length requirements or restrictions in written assignments. The document submitted should be of sufficient length to adequately complete the task. It should not "run off at the mouth." Longer is typically not better. Careful though and editing can shorten and improve an overly long document. Be guided by the following quotation from Blaise Pascal, a very famous mathematician:

> "I have made this letter longer than usual, only because I have not had the time to make it shorter." (Blaise Pascal; French mathematician, physicist, religious philosopher, and child prodigy; 1523–1662.)

On the other hand, it is usually easy to see when a document is overly short—it does not adequately cover the assigned topics of discussion.

This assignment is to write one document that contains (and integrates as seems appropriate):

1. Your educational background, interests, experiences, and professional goals, especially as they might relate to this course. What current access do you have to students as you develop and implement lesson plans related to the material in this course? What grade levels and/or subjects are you teaching
2. Your insights into and prior knowledge about the ideas in the assigned readings and video. Pay particular attention to their possible relevance in your own education and in the education that you are helping students to obtain.

3. What access do your students have to the Web, and what is their experience in making use of the Web and other Information and Communication Technology (ICT)? Share your insights into what your students know about the two-brain and computational thinking ideas covered in the reading and video materials. Feel free to talk with your students about these topics, in order to learn more about what they know.

Unit # 2: Learner Maturity in Math and Other Academic Disciplines

"They know enough who know how to learn." (Henry B. Adams; American novelist, journalist, and historian; 1838–1918.)

"Wisdom is not a product of schooling, but of the life- long attempt to acquire it." (Albert Einstein; German-born theoretical physicist and 1921 Nobel Prize winner; 1879–1955.)

Quick Overview of This Unit

Unit # 1 contained the following brief overview of what one does in studying and learning a discipline:

1. Learn some of the accumulated knowledge of the discipline. The accumulated knowledge is often thought of in terms of data, information, knowledge, wisdom, and foresight. (This is a little confusing, since "knowledge" is used in talking about the totality, and knowledge is also one of the categories on the scale.)
2. Learn to think and solve problems using the accumulated knowledge, tools, and other aids to representing and solving problems in this and other disciplines. For example, reading and writing are tools that cut across all disciplines.
3. Learn about some of the history, culture, beauty, ways of teaching, and ways of learning the discipline.

The overall goals of this course can be analyzed from the point of view of the three components of learning a discipline, especially as applied to math and the sciences. The main focus in the course it on item 2, but there is some emphasis on item 3 in the above list.

In terms of item 1, it is assumed that you already know quite a bit about math content and science content. If this course were a discipline-specific content course, the major goal would be to help you move up the scale in the diagram given below.

General overview of content in a discipline.

However, this is not a math or science content course. **This is a pedagogical content course, with a focus on teaching in a manner that helps students gain in math maturity.** Many of the ideas carry over to helping students gain in science maturity.

There is no universally accepted definition of the level of maturity that a student has achieved in studying in a particular discipline. Moreover, each discipline has its own ideas as to what constitutes "maturity" and how to go about helping students gain an increased level of maturity in the specific academic discipline.

Consider, for example, the discipline of history. Students traditionally think of the study of history as the process of memorizing facts such as dates, names of wars, names of events, names of people, and so on. Contrast this with the following list of four Big Ideas in history education: **causality, investigation, legacy,** and **responsibility.** An increasing level of maturity of a student taking history courses is not measured through a test of memorized facts. Of course, the student needs to have a repertoire of such information. But, increasing maturity is measured by increasing levels of ability to represent, analyze, and think about the problems of history in terms of big ideas such as causality, investigation, legacy, and responsibility. The student is steadily gaining in the ability to read history as an aid to learning history, to draw on multiple sources of information, to interpret conflicting historical stories and records, and so on.

The course you are currently taking uses Math Maturity as a unifying theme. The list of some components of math maturity given below is taken from the required readings of this unit. As you read this list, think about how many of the items are easily modified to apply to science. The items in the list are numbered, but the numbering is not intended to indicate any special ordering of the items. Rather, it facilitates easier reference to particular items in the list. For more information about math maturity, see the Appendix to this book.

1. Communicate mathematics and math ideas orally and in writing using standard notation, vocabulary, and acceptable style.

2. Learn to learn math; complete the significant shift from learning by memorization to learning through understanding. Taking an increasing level of responsibility for your own learning.
3. Transfer one's math knowledge and skills into math related areas and problems in disciplines outside of mathematics.
4. Handle, think about, and think using abstract math ideas. Many students find the level of abstraction introduced in a first year algebra course to be overwhelming. However, students encounter quite a bit of abstraction in math long before they come to an algebra course. For example, compare and contrast: a) One red toy and two blue toys altogether make three toys; and b) $1 + 2 = 3$.
5. Manifest mathematical intuition. Learn to draw on one's accumulated subconscious math knowledge and insights.
6. Move back and forth between the visual (e.g., graphs, geometric representations) and the analytical e.g., (equations, functions).
7. Recognize a valid mathematical or logical argument or proof, and detect 'sloppy' thinking. Provide solid evidence (informal and formal arguments and proofs) of the correctness of one's math work.
8. Recognize mathematical patterns in problems and problem situations that can be represented and analyzed using math.
9. Represent (model) verbal and written word (story) problem situations as mathematical problems.
10. Develop the confidence, knowledge, skills, and persistence to effectively draw upon one's math knowledge and skills to address novel (not previously encountered) challenging math-related problems. Often attempting to deal with novel problem situations requires persistence over a considerable period of time—that is, the ability to deal with delayed gratification. This is a particular challenge to many children who are raised in an environment that tends to foster needs for and satisfaction of instant gratification.

    Note to students: One of the goals in this course is to "stretch" your brain. Ample opportunities will be presented. But, brain stretching is something that you, personally do. Here is an example. Think about why a discussion of science and science literacy typically includes the term "scientific method," while the parallel term "mathematical method" is not used in a discussing math education or math literacy. What is the same and what is different between math and the sciences? What has math contributed to the science disciplines, and what have the science disciplines contributed to math?

The numbered items listed above will be visited and revisited throughout the remainder of this course. As you think about and work with the above list, you will gradually begin to form a personally useful definition of math maturity.

One way to help this process is to think about what the word **maturity** means. A mature adult has the knowledge, skills, attitudes, perseverance, and experience to be a responsible adult citizen in dealing with the types of situations, problems and tasks that occur in the societies and cultures in which he or she lives.

Looking specifically within the discipline of math, a mathematically mature adult has the math knowledge, skills, attitudes, perseverance, and experience to be a responsible adult citizen in dealing with the types of math-related situations, problems, and tasks that occur in the societies and cultures in which he or she lives. In addition, a mathematically mature adult knows when to ask for help, both from other people and from tools. There is no shame in having your income taxes prepared by an expert or for you to make use of income tax preparation software. There is no shame in making use of a financial planning and investment expert.

The level and type of math maturity that adults need vary with what they do. Mathematical maturity for a person who teaches in the K-8 school math curriculum is likely quite different than mathematical maturity for an accountant, artist, sales person in a store, engineer, or musician.

Reading and Video-Viewing Assignment

Moursund, D.G. (2009). Math maturity. Retrieved 5/20/2011 from http://iae-pedia.org/Math_Maturity. Read Parts 1–4 of this Web document.

Moursund, David (2007). Computational thinking and math maturity: Improving math education in K-8 schools. Eugene, OR: Information Age Education. Retrieved from http://i-a-e.org/downloads/doc_download/3-computational-thinking-and-math-maturity-improving-math-education-in-k-8-schools.html. Read from the beginning up through page 29 of this book.

Graded Assignment for Unit # 2

Assignment # 2 (100 points). All assignments are to be submitted as attachments to an email message sent to the course instructor. The assignment should be done using Microsoft Word or some other commonly used word processing system that includes a spelling check and sufficient desktop publishing capabilities to do an acceptable level of desktop publication of the document. It is assumed that submitted documents will have been carefully spell checked and be of a writing quality that one expects of graduate students.

This assignment is to write one document that contains (and integrates as seems appropriate):

1. Your agreements with and disagreements with the ideas about math maturity that are presented in the assigned readings.
2. Your personal insight on what you look for in determining the math maturity and/or science maturity of students you teach. What do you look for in talking with the student, observing a students classroom behavior, listening to the student answering questions in class, being a responsible student, and so on? In this discussion it is appropriate to site example that illustrate low and high levels of maturity. Keep in mind that maturity is not the same as knowledge of specific topics in a discipline.
3. Select one or two students you are teaching who appear to have a below average level of math maturity and one or two who appear to have an above average level of math maturity. Carry on an individual conversation with each that is designed to give you an increased level of insight into what constitutes math maturity and how one can gain information about a student's level of

math maturity. Your write-up on this activity should include a compare a compare and contrast. It should be designed to help your course instructor learn some of what you already know and some of that you learned through this activity with your students. (Assume that your course instructor has had little or no experience in working with the age range, grade level, and types of students you are currently working with.)

Unit # 3: Personal Electronic Digital Filing Cabinet

"The strongest memory is not as strong as the weakest ink." (Confucius; Chinese thinker and social philosopher, whose teachings and philosophy have deeply influenced Chinese, Korean, Japanese, Taiwanese and Vietnamese thought and life; 551 BC–479 BC.)

"Consider a future device for individual use, which is a sort of mechanized private file and library. It needs a name, and, to coin one at random, 'memex' will do. A memex is a device in which an individual stores all his books, records, and communications, and which is mechanized so that it may be consulted with exceeding speed and flexibility. It is an enlarged intimate supplement to his memory." (Vannevar Bush; *As We May Think,* The Atlantic, July 1945; American engineer and science administrator known for his work on analog computing, his political role in the development of the atomic bomb, and the idea of the memex, which was seen decades later as a pioneering concept for the World Wide Web; 1890–1974.)

Quick Overview of This Unit

All teachers collect "good stuff" that they make use of in their teaching and in their own learning. Some of this lends itself to being stored on shelves or in boxes and bins. Other parts are paper that might be stored in a physical filing cabinet or as electronic documents in an electronic digital filing cabinet. Audio, video, and photographs can be stored as physical items or in electronic digital formats.

Most likely you have both a personal physical library and a good start on a personal electronic digital library. Your electronic digital library may contain copies of email that you have sent or received, copies of papers that you have written for classes, handout and worksheets you use in teaching, and so on. It may contain links to audio and video recordings available on the Web, and it might include your personal electronic digital copies of various audio and video materials.

You may well think of the school library or local public or university as a supplement to and extension of your personal physical library. And, of course, you can think of the Web as a supplement to and extension of your electronic digital library. Your personal physical and electronic digital libraries contain content that you have personalized (for example, by creating and/or reading and/or using with your students, etc.) It may well contain links to documents on the Web or accessible from servers you can access. But, a personal electronic digital filing cabinet is in no sense just a bunch of links to and/or copies of stuff that you have not read and have little or no personal involvement with.

This unit focuses on the idea of a personal professional electronic digital filing cabinet. A somewhat parallel idea is that of students having a personal academic electronic filing cabinet and developing an electronic portfolio. Developing and maintaining a personal math electronic filing cabinet can be one aspect of indicating an increasing level of math maturity.

An electronic digital filing cabinet is designed as a supplement to one's brain. You may have already designed a math EDFC that fits your personal needs, and stored quite a bit of information in it. Or, it may be that you have not yet done this, and the assignment given here will be your starting point.

In either case, think about accumulating information that you will want to retrieve and use sometime in the future, and accumulating information that you may want to share with others. Quite likely you will find that a good design has features such as:

1. It is easy to keep the original copy and one or more back-up copies in different locations.
2. It is easy to search, add items, edit items, and delete items.
3. Some of the entries are complete documents—for example, complete documents you have written, complete copies of documents that you have received from others, worksheets and other handout material used in teaching, and so on.
4. Some of the entries will be pointers to or links to other documents. You will likely find that it is helpful to include complete citations in some generally accepted citation style. In addition, if you provide a link to a Website, include a brief description of the content. Over time, many links become broken. But, from a brief summary of the contents, you can use a search engine to find the original document or to find other documents that contain roughly the same information.

Reading and Video-Viewing Assignment

Moursund, D.G. (n.d.). Digital filing cabinet: Overview. Retrieved 5/20/2011 from http://iae-pedia.org/Digital_Filing_Cabinet/Overview.

Moursund, D.G. (n.d.). Math education digital filing cabinet. Retrieved 5/20/2011 from http://iae-pedia.org/Math_Education_Digital_Filing_Cabinet.

Moursund, D.G. (n.d.). Help section of the Information Age Education Wiki. Retrieved 5/20/2011 from http://iae-pedia.org/Help:Contents. Also see the Help menu item on the left side of IAE Wiki pages.

In addition, make yourself aware of the following resources. That is, do a quick browse of these resources—staying at a site long enough to make a reasoned decision as to whether you might want to make use of the content sometime in the future.

Moursund, D.G. (n.d.). Math Education Quotations. http://iae-pedia.org/Math_Education_Quotations,

- Math Education Free Videos. http://iae-pedia.org/Math_Education_Free_Videos.
- No Cost Educational Videos. http://iae-pedia.org/No_Cost_Educational_Videos.
- Open Source Textbooks. http://iae-pedia.org/Open_Source_Textbooks.
- Open Content Libraries. http://iae-pedia.org/Open_Content_Libraries.
- Free Math Software. http://iae-pedia.org/Free_Math_Software.

- Open Source Software Packages. http://iae-pedia.org/Open_Source_Software_Packages.

Graded Assignment for Unit # 3

Assignment # 3 (100 points). All assignments are to be submitted as attachments to an email message sent to the course instructor. The assignment should be done using Microsoft Word or some other commonly used word processing system that includes a spelling check and sufficient desktop publishing capabilities to do an acceptable level of desktop publication of the document. It is assumed that submitted documents will have been carefully spell checked and be of a writing quality that one expects of graduate students.

This assignment is to write one document that contains (and integrates as seems appropriate):

1. What you have previously done (before starting this course) in terms of collecting and/or creating hard copy and electronic documents that might be relevant to your teaching and your personal learning. Include a description of how your collection of documents is organized, how well it meets your needs and the security precautions you take—for example, against unauthorized access or against documents becoming lost, damaged, or destroyed.

   1a. If you don't already have a math EDFC that is appropriately designed to meet you needs, start one. If you already have a math EDFC, do whatever modifications or redesign work you feel is appropriate to better fit your needs.

   1b. Throughout the remainder of this course, the expectation is that you will make regular additions to your math EDFC. For each addition that is specifically relevant to this course, make sure that the entry contains a citation in an acceptable style (including a link if appropriate), date added, and a short paragraph describing the content and its relevance to this course. The expectation is that you will make several such additions as part of your studies for each of the remaining Units. Unit 10 includes a 50-point assignment requiring you to submit an electronic copy of these citations and short summaries.

   1c. **Do not** turn in these new additions to your EDFC in a piecemeal fashion. Save all of them until the end of the course and turn them as part of the document that is Assignment # 10 of the course.

2. Many of the required readings for this course come from the Information Age Education Wiki and were written by David Moursund. Select a document in the Information Age Education Wiki that seems to you to be relevant to this course. Make a contribution either to the document itself or to its accompanying Discussion (assuming it has one—if it doesn't, feel free to add one to the Wiki). To do this, you will need to log onto the site, (See the third of the required readings for this unit.) The written assignment you turn in should include a brief discussion of the purpose or intent of your addition and an easily followed link or set of instructions that your instructor can follow to find your contribution in the Information Age Education Wiki.

Unit # 4: Nature and Nurture Aspects of Brain Science

> "... pedagogy is what our species does best. We are teachers, and we want to teach while sitting around the campfire rather than being continually present during our offspring's trial-and-error experiences." (Michael S. Gazzaniga; professor of psychology and a brain scientist at the University of California, Santa Barbara, where he heads the new SAGE Center for the Study of the Mind 1939–.)

Quick Overview of This Unit

An intact human brain is naturally curious and has a tremendous capacity to learn. Its plasticity allows it to deal with some injuries and to continue to learn throughout one's lifetime.

People are born with varying types and levels of innate capabilities. In terms of IQ, for example, studies of identical twins suggest that nature (innate capacity) and nurture (informal and formal education and experiences; damage due to injuries, poisons, drugs, malnutrition, and so on) each contribute a great deal to a to a person's developing intelligence. For many years, researchers have studied human intelligence and human cognitive development. The names Alfred Benet (IQ) and Jean Piaget (cognitive development) are likely familiar to you. During the past two decades, brain-scanning equipment of various sorts has contributed immensely to the discipline of brain science (cognitive neuroscience).

The basic ideas of math maturity are rooted in intelligence, cognitive development, and in the discipline of math. In somewhat overly simplistic terms, a student's level of math maturity can be increased through helping the student develop and make use of his or her general intelligence and math intelligence, general and math cognitive development, and math knowledge, skills, interest, and experience. This three-part approach—intelligence; cognitive development; and domain specific knowledge, skills, interest, and experience—holds for studying and building maturity in any discipline.

Some people learn more, better, and faster than others. In a typical K-8 school classroom there may be one or two students who learn math less than half as fast as the class average, and one or two who learn math more than twice as fast as the class average. Both nature and nurture contribute to this situation.

Math is taught in a vertically structured manner, with content at a higher grade level being build on the math knowledge and skills that students supposedly learned and retained from earlier. At any grade level, the wide variation in students presents a major challenge to the teacher and the students.

Take another look at the list about learning in a discipline that has been presented twice before in this Syllabus:

1. Learn some of the accumulated knowledge of the discipline. The accumulated knowledge is often thought of in terms of data, information, knowledge, wisdom, and foresight. (This is a little confusing, since "knowledge" is used in talking about the totality, and knowledge is also one of the categories on the scale.)
2. Learn to think and solve problems using the accumulated knowledge, tools, and other aids to representing and solving problems. In this and other

disciplines. For example, reading and writing are tools that cut across all disciplines.

3. Learn about some of the history, culture, beauty, ways of teaching, and ways of learning the discipline.

Our educational system, and each individual teacher, is faced by the issue of how much teaching and learning effort should be focused on each of these components.

This challenge becomes even greater in dealing with slow learners and students with various learning disabilities. How should the learning capabilities and time of such students be best used to help them achieve a level of math maturity that will serve them in adulthood?

Reading and Video-Viewing Assignment

1. Fischer, Kurt. Three short videos of Fischer presenting his insights into brain science and education. Harvard Graduate School of Education. Retrieved 5/20/2011 from : http://www.uknow.gse.harvard.edu/learning/video-learn002b-uk_kf_growthcyc.html. Overview. (3:20). Brain Growth (2:09). Cognitive Development (2:36). Each script is followed by a video that requires Real Player. Real Player is free software. There are a Mac and a PC version. See the Download Now button near the bottom of the page: http://www.player-media.net/realplayer_sp/index.php?source=CCN-CD282-GOOG1326-US-real1&googleid=1029811326.

2. Sternberg, Robert (2007). Interview with Dr. Sternberg. Retrieved 5/20/2011 from http://www.indiana.edu/~intell/sternberg_interview.shtml. Includes both video interview and script.

3. deCharmes, Christopher (February 2008). Christopher deCharms looks inside the brain. (4:00). Retrieved 5/20/2011 from http://www.ted.com/talks/christopher_decharms_scans_the_brain_in_real_time.html. Provides insight into the frontiers of brain science and progress in highly individualizing instruction of one's own brain.

4. (**Just for fun. This is not a required video viewing. However, you will likely enjoy the video and learn some things about the brain that are relevant to education and that you have not previously encountered.)** Ramachandran, Vilayanur (March 2007). Ramachandran on your mind. 23:38 video. Retrieved 5/20/2011 from http://www.ted.com/talks/vilayanur_ramachandran_on_your_mind.html. This talk provides some insight into how the brain works and how brain science research is done.

   This is one of a large and growing collection of Technology Entertainment Design (TED educational videos

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5. .

6. Parts 5, 6, and 7 of Math Maturity at http://iae-pedia.org/Math_Maturity.

7. **(This is not a required reading. The following free book may be of particular interest to some of the people n this course.)**

Moursund, David (2007). Computers in education for talented and gifted students: A book for elementary and middle school teachers. Eugene, OR: Information Age Education. Access at http://i-a-e.org/downloads/doc_download/13-computers-in-education-for-talented-and-gifted-students.html.

Graded Assignments

The readings for this unit provide a summary of some of what is known about intelligence and IQ, some of what is know about a stage theory of cognitive development, and some of what is known about both relatively slow and relatively fast learners of math.

Assignment # 4 (100 points). All assignments are to be submitted as attachments to an email message sent to the course instructor. The assignment should be done using Microsoft Word or some other commonly used word processing system that includes a spelling check and sufficient desktop publishing capabilities to do an acceptable level of desktop publication of the document. It is assumed that submitted documents will have been carefully spell checked and be of a writing quality that one expects of graduate students.

This assignment is to write one document that contains (and integrates as seems appropriate) material from this unit with material from previous units. Begin by thinking about the very bottom and the very top of the math (or, if you like, science) students you are teaching. Do a compare and contrast discussion of these two categories of students that is based on the ideas presented in the required readings and video. (If you would find it helpful, you might want to do an informal interview of one or two students from each category.) Next, analyze what you are currently doing to help increase the level of math maturity of each of these two groups. What is working and what isn't working as well as you would like? Again, a compare and contrast analysis would be appropriate.

There are two major goals in this assignment:

1. Show that you understand the materials in the assigned readings and video viewings.
2. Explore various possible relevance of this material in terms of teaching for increased math (or science) maturity of your students, paying special attention to the two extremes of students you work with.

In addition, note that Assignment #10 (the last assignment in this course) is partly based on the assumption that you are making additions to you personal professional Electronic Digital Filing Cabinet on a regular basis. Please **do not** provide comments on that activity as you turn in each assignment. Instead, at the time you submit an assignment, think about the assigned and other readings and video viewings you have done since submitting the previous assignment. Based on this, consider making one or more additions to your EDFC. Remember that entries in your EDFC should be "stuff" for which you have some personal ownership and interest. If can be materials specifically useful to your professional development, materials that you might want to share with other educators, and materials you want to use in your teaching.

Unit # 5 Math Problem Solving Across the Curriculum

" Mathematics consists of content and know-how. What is know-how in mathematics? The ability to solve problems.” (George Polya; born in Hungary; long career as a mathematician in Switzerland and the United States; 1887–1985.)

"Each problem that I solved became a rule which served afterwards to solve other problems." (René Descartes; French philosopher, mathematician, scientist, and writer; 1596–1650.)

Quick Overview of This Unit

Problem solving: Moving from A to B.

One good sign of a person’s increasing level of maturity in a discipline is the person’s increasing level of problem solving knowledge, skills, and thinking that cut across that discipline and its applications in other disciplines.

Problem solving is part of each academic discipline. Problem solving tends to require a combination of general knowledge and skills that are domain independent, and knowledge and skills quite specific to one or more domains. Here is a quote from a very famous scientist:

> "I believe that a scientist looking at nonscientific problems is just as dumb as the next guy." (Richard Feynman; Nobel Prize winning American physicist; 1918–1988.)

As physicist, Richard Feynman developed knowledge and skills in physics and mathematics that put him at the forefront of his profession. However, this does not automatically mean he was skilled at art, chess, economics, or psychology. Still, the ways that scientists and mathematicians view the world and think about problem solving are useful in many different disciplines.

Here is a somewhat different way of looking at this situation. An increasing level of maturity in a discipline such as math or science is evidenced by learning to think like a mathematician or scientist. This “thinking like” characteristic can be applied in any discipline. A mature artist views the world through “artist-colored glasses.” A mature poet sees poetry everywhere.

Quoting from the Wikipedia (http://en.wikipedia.org/wiki/Problem_solving):

> Problem solving forms part of thinking. Considered the most complex of all intellectual functions, problem solving has been defined as higher-order cognitive process that requires the modulation and control of more routine or fundamental

skills (Goldstein & Levin, 1987). It occurs if an organism or an artificial intelligence system does not know how to proceed from a given state to a desired goal state. It is part of the larger problem process that includes problem finding and problem shaping.

Problem solving includes:

- **Question situations:** recognizing, posing, clarifying, and answering questions.
- **Problem situations:** recognizing, posing, clarifying, and then solving problems.
- **Task situations:** recognizing, posing, clarifying, and accomplishing tasks.
- **Decision situations:** recognizing, posing, clarifying, and making good decisions.
- **Thinking:** using higher-order critical, creative, wise, and foresightful thinking to do all of the above. Often the results are shared, demonstrated, or used in a product, performance, or presentation.

Here is a definition of problem that fits well in many different disciplines. You (personally) have a problem if the following four conditions are satisfied:

1. You have a clearly defined given initial situation.
2. You have a clearly defined goal (a desired end situation). Some writers talk about having multiple goals in a problem. However, such a multiple goal situation can be broken down into a number of single-goal problems.
3. You have a clearly defined set of resources that may be applicable in helping you move from the given initial situation to the desired goal situation. These typically include some of your time, knowledge, and skills. Resources might include money, the Web, and the telecommunication system. There may be specified limitations on resources, such as rules, regulations, guidelines, and time lines for what you are allowed to do in attempting to solve a particular problem.
4. You have some ownership—you are committed to using some of your own resources, such as your knowledge, skills, time, and energy, to achieve the desired final goal.

Reading and Video-Viewing Assignment

1. Robinson, Ken (2006). Ken Robinson says schools kill creativity. (19:21 video.) Retrieved 5/20/2011 from http://www.ted.com/talks/ken_robinson_says_schools_kill_creativity.html. Creativity expert Sir Ken Robinson challenges the way we're educating our children. He champions a radical rethink of our school systems, to cultivate creativity and acknowledge multiple types of intelligence. As you view this video, think abut the fifth of the bulleted items given earlier: "**Thinking:** using higher-order critical, **creative,** wise, and foresightful thinking to do all of the above. Often the results are shared, demonstrated, or used in a product, performance, or presentation."
2. Babbage's Mechanical Calculator Comes to Life. (Four-minute video demonstrating a machine designed to help solve the math problem of creating

accurate math table.) Retrieved 5/20/2011from http://www.wired.com/gadgetlab/2008/05/exclusive-video/. Think about what aspects of solving a problem can be done by a non-thinking machine and what aspects require creative use of one's brain.

3. Moursund, D.G. (2007). Introduction to problem solving in the Information Age. Eugene, OR: Information Age Education. Access at http://i-a-e.org/downloads/doc_download/7-introduction-to-problem-solving-in-the-information-age.html. This short (99 page) book is intended primarily for preservice and inservice teachers of K-12 students, and the teachers of these teachers. In this book, the term problem solving includes posing and solving problems, posing and accomplishing tasks, posing and answering questions, and posing and making decisions.

Specific reading assignment: Read the Preface, Chapters 1-3. and Chapter 5

Graded Assignment

Here is a brief summary of the readings and video viewing for this unit. It provides the background for the assignment. There are three key ideas in this unit.

- A broad definition of problem solving.
- A definition of "problem" that fits reasonably well with problems in most disciplines.
- A focus on learning to think in a discipline.

Starting at the earliest levels of formal education, teachers can orient their students in these directions and help students increase their levels of conscious understanding of these ideas. Obviously, what can be accomplished at each grade level depends on the individual students and what has been accomplished at earlier grade levels. Equally obviously, the teacher and the overall problem-solving orientation of the classroom environment make a huge difference.

Identifying, creating, understanding, and solving problems is part of every academic discipline, and thus runs throughout the curriculum. If one encounters a particular problem and slight variations of the problem over and over again, then it makes sense to memorize how to solve the problem. With practice one gains speed and accuracy so that, in essence, the problem is solve automatically with little or no conscious thought when it is encountered.

However, there are far too many problems that one may encounter in life to make it feasible to study each one through this rote memory approach.

Thus, a good education for problem solving has a an appropriate balance between learning to rapidly and accurately solving a set of frequently occurring problems, and learning to deal with less frequently occurring problems.

The list of various aspects of increasing math maturity given in Unit 2 includes: "Learn to learn math; complete the significant shift from learning by memorization to learning through understanding." Remember, this same concept of an increasing level holds in every academic discipline.

Assignment # 5 (100 points). All assignments are to be submitted as attachments to an email message sent to the course instructor. The assignment should be done using Microsoft Word or some other commonly used word processing system that includes a spelling check and sufficient desktop publishing capabilities to do an acceptable level of desktop publication of the document. It is assumed that submitted documents will have been carefully spell checked and be of a writing quality that one expects of graduate students.

This assignment is to write one document that contains (and integrates as seems appropriate) the key ideas from the reading and video assignment with what you and your students are actually doing. In essence, you are to observe yourself and your students and do a needs assessment. What is actually happening versus what you and your students are capable of making happen?

One possible starting point is to spend a couple of days observing yourself and your students. Look for behaviors, teaching activities, and learning activities that illustrate moving from a rote memory approach in math (or, in math and science) learning and problem solving to a understanding approach to learning and problem solving. Also, look for learning difficulties that some individual students are experiencing that suggest the learning demands are well over the cognitive development of the student. (That is, look for situations in which students lack the math and/or science maturity to deal with the types of problems they are encountering.)

Next, analyze what you are observing versus the three major ideas in the readings. Do the ideas seem to have "face validity?" That is, do they make sense to you and the teaching situations you face? What makes sense and what does not make sense? Why?

Finally, begin to introduce and to try out the three main ideas with your students—at a level that is appropriate to your students. What do they currently know about these three ideas? What are you doing to help them gain explicit knowledge about the ideas and to increase their understanding and use of the ideas? What can you be doing? Do a little experimentation and report on the results.

This assignment is designed to stretch your pedagogical content knowledge and teaching skills. It is designed to help increase your level of math and/or science pedagogical maturity.

In addition, note that Assignment #10 (the last assignment in this course) is partly based on the assumption that you are making additions to you personal professional Electronic Digital Filing Cabinet on a regular basis. Please **do not** provide comments on that activity as you turn in each assignment. Instead, at the time you submit an assignment, think about the assigned and other readings and video viewings you have done since submitting the previous assignment. Based on this, consider making one or more additions to your EDFC. Remember that entries in your EDFC should be "stuff" for which you have some personal ownership and interest. If can be materials specifically useful to your professional development, materials that you might want to share with other educators, and materials you want to use in your teaching.

Unit # 6: Math Word Problems

> "An individual understands a concept, skill, theory, or domain of knowledge to the extent that he or she can apply it appropriately in a new situation." (Howard Gardner, The Disciplined Mind: What All Students Should Understand, Simon & Schuster, 1999.)

> "In the book of life, the answers aren't in the back." (Charles Schulz; American cartoonist best known worldwide for his Peanuts comic strip; the quoted statement is from the comic strip character Charlie Brown; 1922–2000.)

Quick Overview of This Unit

Word problems or "story" problems are a well-established component of the math education curriculum. Indeed, some students and math teachers tend to equate "word problem" with "problem" when talking about problem solving in math. Moreover, many people seem to think that math is the only part of the school curriculum that teaches problem solving.

However, problems and problem solving are part of every academic discipline. Each discipline develops some special vocabulary, symbols, and methods for representing and communicating its problems. Thus, word problems are an important part of each discipline of study. Each academic discipline develops methods for accumulating and storing information. Each discipline develops methodologies—educators call it content pedagogical knowledge—useful in helping students learn the discipline. All of these activities involve oral and written communication.

One measure of a student's increasing level of math maturity is the student's increasing ability to communicate effectively—speak, listen, read, write, all with understanding—in the language of mathematics.

This unit focuses on word problems in math. The reading identifies a number of characteristics of "good" word problems and of good teaching that accompanies use of such word problems. "Good" tends to be based on how the problems and the teaching contribute to increasing student levels of math maturity and their thinking approaches to problem solving.

You are undoubtedly familiar with the traditional types of word problems used in math books. Likely you have also encountered logic puzzles or brain teasers. Here is an example:

> At Parkview High School, the science club has 11 members, the computer club has 14 members, and the puzzle club has 25 members. If a total of 15 students belong to only one of the three clubs, and 10 belong to only two of them, how many students belong to all three clubs?

Notice that this is a type of logic puzzle that requires significant reading skill. Many brain teaser puzzles require good reading skills and good use of logic. In addition, math skills are often helpful. Math people can solve this particular math puzzle mentally, using only elementary school arithmetic. If you are not able to figure out a direct way to solve the puzzle, think about using trial and error—perhaps in conjunction with a Venn diagram picture of the situation.

Reading and Video-Viewing Assignment

1. Albrecht, Robert and D.G. Moursund (October 2009). Word problems in math. Retrieved 5/20/2011 from http://iae-pedia.org/Word_Problems_in_Math.
2. Moursund, D.G. and Ricketts, Dick (n.d.). Communication in the language of mathematics. Retrieved 5/20/2011 from http://iae-pedia.org/Communicating_in_the_Language_of_Mathematics.

Graded Assignment

Assignment # 6 (100 points). All assignments are to be submitted as attachments to an email message sent to the course instructor. The assignment should be done using Microsoft Word or some other commonly used word processing system that includes a spelling check and sufficient desktop publishing capabilities to do an acceptable level of desktop publication of the document. It is assumed that submitted documents will have been carefully spell checked and be of a writing quality that one expects of graduate students.

This assignment has two parts:

1. A recent Google search on *free online brain-teaser game* produced over 500,000 hits. For example, the Website http://www.puzz.com/iqteasers.html contains a number of what it calls IQ Brain Teasers. Many sites contain word problems, with the quality varying tremendously. For example, see http://www.kangurusa.com/clark/pdb/ and http://www.mathplayground.com/wpdatabase/wpindex.html. (The examples given in this paragraph were not chosen for the quality of the brain teasers and word problems. Rather, they were chosen because the sites are reasonably user friendly and contain a number of problems.

   Spend some time browsing the Web to find brain teasers and math word problems that have the dual characteristics: a) they are at a level that challenges students in your class; b) they have some of the characteristics of "good" math word problems as described in the required readings.

   Designate a piece of your bulletin board as the Problem of the Week. (After you have accumulated a lot of problems, you can change this to the Problem of the Day.) Each week (or, each day, or twice a week, etc.) post a math word problem or a logical (brain teaser) puzzle that is suited to the level of students you teach. Leave a problem or puzzle posted for perhaps several days or a week. A good problem of puzzle has some of the characteristics of a good work problem (see the reading assignment that contains that topic) and is a challenge to students. Since there are lots of such problems and puzzles available on the Web, you can also provide your students with the challenge of adding to your bulletin board. Students who find solutions can write them up and submit them to you, or can explain them to you orally.

   The specific assignment is to make a "good faith" start on this activity and write about the results. Your analysis should provide your insights as to whether this can be a useful part of your teaching for increased math maturity.

In addition, use this activity to add to your Math Electronic Digital Filing Cabinet.

2. Many middle school and high school math teachers report that their students are poor at dealing with math word problems and strongly dislike —perhaps even "hate"—math word problems. As with many educational problems, it is common to place blame on the teachers that the students have previously had. The intent in this activity is for you to do an analysis of the nature and extent of teaching about and making use of math word problems (and/or word problems that include a combination of math and science) that you use in your teaching.

   You might want to approach this as a three-part activity. First you observe and do a needs assessment. Second, you decide on some intervention based on the needs assessment and your insights from the assigned readings. Third you report on the results. The intervention of a Problem of the Week does not count as the intervention to be used in this piece of the assignment.

In addition, note that Assignment #10 (the last assignment in this course) is partly based on the assumption that you are making additions to you personal professional Electronic Digital Filing Cabinet on a regular basis. Please **do not** provide comments on that activity as you turn in each assignment. Instead, at the time you submit an assignment, think about the assigned and other readings and video viewings you have done since submitting the previous assignment. Based on this, consider making one or more additions to your EDFC. Remember that entries in your EDFC should be "stuff" for which you have some personal ownership and interest. If can be materials specifically useful to your professional development, materials that you might want to share with other educators, and materials you want to use in your teaching.

In addition, note that Assignment #10 (the last assignment in this course) is partly based on the assumption that you are making additions to you personal professional Electronic Digital Filing Cabinet on a regular basis. Please **do not** provide comments on that activity as you turn in each assignment. Instead, at the time you submit an assignment, think about the assigned and other readings and video viewings you have done since submitting the previous assignment. Based on this, consider making one or more additions to your EDFC. Remember that entries in your EDFC should be "stuff" for which you have some personal ownership and interest. If can be materials specifically useful to your professional development, materials that you might want to share with other educators, and materials you want to use in your teaching.

Unit # 7: Communication in Math

"A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas."(G.H. Hardy; prominent English mathematician, known for his achievements in number theory and mathematical analysis; 1877–1947.)

"God created the natural numbers. All the rest is the work of man." (Leopold Kronecker; German mathematician and logician; 1823-1891.)

Quick Overview of This Unit

This Unit focuses on helping students get better at reading, writing, speaking, listening, and thinking in the language of math.

We know quite a bit about a child leaning to communicate orally and think in a natural language such as English. Under appropriate circumstances, a child readily becomes bilingual or trilingual in oral language.

We know most children find it to be a significant challenge to learn to read and write a natural language in which they already have oral fluency. In this learning endeavor, students already know how to think in the natural language. If the written language is phonetic or somewhat phonetic, this is also a major help toward learning to read the language.

Now, consider the problems that children face in learning to communicate in and think in mathematics. Most students facing this task have relatively little oral fluency in math and their thinking skills in math are only modestly developed. While some of the words in math come from the "natural" language in which students have oral communicant skills, the math versions of these words tend to take on new, more precise meaning, and many of the math symbols or words are not phonetic.

As a simple example, think about the number sentences:

$2 + 5 = 7$.

$(5 + 4)/3 - 1 = 2$.

$4 + 2 < 7$.

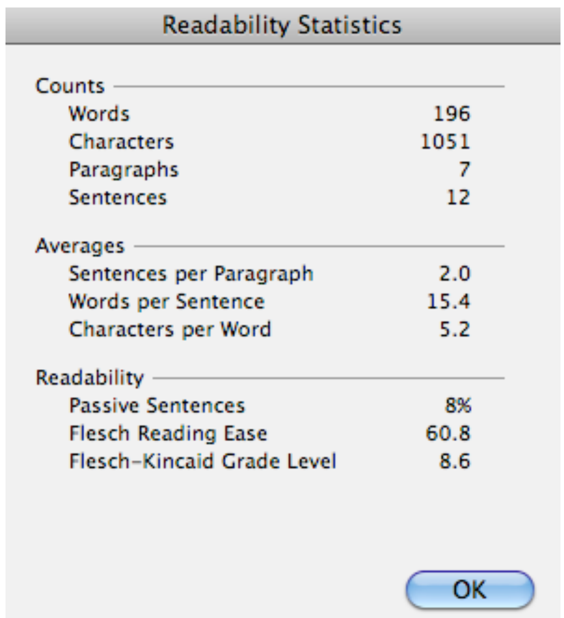
Gains in oral and written math communication skills, and learning to think in the language of math, are important aspects of increasing math maturity. Some possible approaches to increasing these aspects of math maturity include:

1. Teacher doing role modeling. This is a traditional way of teaching math, with teachers "talking and demonstrating." This "chalk and talk" approach allows the teacher to also "think out loud" to help students gain insight into what is going on.
2. Students orally explaining their math work to the whole class and students doing work at the chalkboard or white board.

3. Students writing explanations and justifications for the steps they take in solving an assigned seatwork or homework problem.
4. Students talking to each other about math—for example, explaining what they are doing and providing help to each other.
5. Students doing math journaling and other types of math-related writing, such as crating word problems for other students to solve.
6. Students being explicitly taught to read math materials and learn from these written math materials. Help students learn to read and understand their math books. Help students learn to read and understand math word problems. Help students learn to create math word problems.

Readability

You have probably encountered various measures of the reading level of materials. Here is what Microsoft Word reports on the last paragraph (with its numbered list) in the previous section.



Books for beginning readers tend to have carefully restricted vocabulary, lots of pictures, short sentences, and lots of repetition. Many readability formulas have been developed. (See http://en.wikipedia.org/wiki/Readability_formula.)

There are a variety of readability tests available on the Web, and some word processors include built-in readability tests. For example, Microsoft Word reports that the Flesh-Kincaid Grade Level for this whole Syllabus in about 10.8.

However, such formula-based approaches to measuring readability level do not work well on math textbooks. For example, the challenge of reading, understanding, and solving a math word problem is not accurately estimated by its Flesch-Kincaid Grade Level or other simple formula-based measure of readability.

The issue is comprehensibility. What sort of math text might we expect an average fifth grade student to be able to read and comprehend? Obviously this depends not only on the general level of reading knowledge and skills the student has, but also on specific math-reading knowledge, skills, and experience.

Reading a math book has some of the same challenges as reading math word problems. Both the math book and the math word problem present ideas and problem situations using natural language. Math books tend to make considerable use of math symbols and math vocabulary. From that point of view, we might guess that it is harder to learn to read a math book than it is to learn to read just the math word problems that appear in the book.

On the other hand, a math word problem tends to be rather short and succinct. The density of its content and ideas may make it a greater challenge to read with understanding than a math textbook. Moreover, the math word problem may well be presented without any contextual information and examples, while the textbook provides a discussion of the math situation it is presenting and some examples.

These ideas have long been known. The following is quoted from Dubris and Merchant (1993):

> According to Joan Curry, "Mathematics is a highly condensed system of language." Because reading is mathematics involves not only decoding words but also attaching literal meaning to mathematical symbols, and discerning the relationship between the two, teaching reading in the content area of mathematics is a particularly challenging task.
>
> Further complicating the integration of the two disciplines is the usual practice of teaching reading skills and processes separate from mathematics. Thus, the transfer from reading class to mathematics is difficult but essential.
>
> An especially difficult problem faced by students is that the reading level normally associated with a particular grade is often lower than the level necessary to comprehend that grade's mathematics text. Even the best readers can have difficulty making the transition from a narrative text, with its plot, characters, and setting, to a content-area expository text with a hierarchical pattern of main ideas and supporting details. Math texts are written in an especially terse and unimaginative style; they offer few verbal context clues to help with decoding meaning; and they lack the redundancy that makes writing easier to read. Another complicating factor is the variety of eye movements required to read math. The left-to-right rule often does not apply in reading number operations, as the set of diagrams on page 65 makes graphically clear.
>
> The classic example of reading pitfall in math class is the "word problem." Students of mathematics who have not previously developed the necessary reading skills might be able to the arithmetic if only they could read the problem with understanding.

Reading and Video-Viewing Assignment

1. Moursund, D.G. and Ricketts, Dick (n.d.). Communicating in the language of mathematic. Retrieved 5/20/2011 from http://iae-

pedia.org/Communicating_in_the_Language_of_Mathematics. Here is a very brief summary of this Web document:

> Communication in math involves making use of processes of reading, writing, speaking, listening, and thinking as one communicates with one's self, other people, computers, books, and other aids to the storage, retrieval, and use of the collected mathematical knowledge of the world. Current precollege math education systems have substantial room for improvement in helping students learn to communicate effectively in the "language" of mathematics.

Graded Assignment

Assignment # 7 (100 points). All assignments are to be submitted as attachments to an email message sent to the course instructor. The assignment should be done using Microsoft Word or some other commonly used word processing system that includes a spelling check and sufficient desktop publishing capabilities to do an acceptable level of desktop publication of the document. It is assumed that submitted documents will have been carefully spell checked and be of a writing quality that one expects of graduate students.

This assignment is to write one document that contains (and integrates as seems appropriate) the key ideas from the reading and video assignment with the following questions and activities.

1. Analyze the way you teach math at a particular grade level. Focus on the idea of teaching using "oral tradition" versus teaching in a manner that makes use of students' math reading and writing knowledge and skills and is designed to help your students learn to read and write math.
2. The material (quoted earlier in Unit 7) from the Dubris and Merchant (1993) suggests that math books tend to be written at a significantly higher grade level of reading than the Language Arts books used at the same grade level. Discuss this assertion, mainly by making use of your personal experiences in teaching math. (If you like, you can also do a literature search in this area and see if you can find recent research on the topic.)
3. Locate about four of five typical (representative) math word problems from a math text designed for use by students at a particular grade level. (If you teach math at the K-2 level, you will likely need to use material and students from a higher grade level in order to complete this assignment.) Determine their grade level reading level difficulty by use of some standard formula-based method. Then analyze these questions from the following points of view:
    a. There "goodness" according to the criteria in the previously required reading on Word Problems in Math at (retrieved 5/20/2011) http://iae-pedia.org/Word_Problems_in_Math.
    b. The nature and extent of the types of difficulties and successes you believe students would have in attempting to solve these problems.
4. Continuing to use the word problems from (3.). Try these problems out with students and observe the results. This can be done with a whole class or it can be done by your direct observation in a one-on-one setting as a quite small number of students try the problems. In this latter case, try to use "average"

students and perhaps have them “think out loud” as they work on the problems.

In addition, note that Assignment #10 (the last assignment in this course) is partly based on the assumption that you are making additions to you personal professional Electronic Digital Filing Cabinet on a regular basis. Please **do not** provide comments on that activity as you turn in each assignment. Instead, at the time you submit an assignment, think about the assigned and other readings and video viewings you have done since submitting the previous assignment. Based on this, consider making one or more additions to your EDFC. Remember that entries in your EDFC should be “stuff” for which you have some personal ownership and interest. If can be materials specifically useful to your professional development, materials that you might want to share with other educators, and materials you want to use in your teaching.

Unit # 8: Good Math Lesson Plans

"A great teacher makes hard things easy." (Ralph Waldo Emerson; American essayist, philosopher, poet, and leader; 1803–1882.)

"The point is to make math intrinsically interesting to children. We should not have to sell mathematics by pointing to its usefulness in other subject areas, which, of course, is real. Love for math will not come about by trying to convince a child that it happens to be a handy tool for life; it grows when a good teacher can draw out a child's curiosity about how numbers and mathematical principles work. The very high percentage of adults who are unashamed to say that they are bad with math is a good indication of how maligned the subject is and how very little we were taught in school about the enchantment of numbers." Alfred Posamentier; Professor of Mathematics Education at the City College of New York; 2002 New York Times article.

Quick Overview of This Unit

The following is quoted from the first part of the assigned readings.

> "Lesson plan" usually refers to a single lesson, designed for one class period. However, it can also refer to a sequence of such plans designed for a unit of study. (Such a sequence may be called a unit plan.) In this document, "lesson plan" mean a plan to facilitate one more class periods of organized teaching and learning.
>
> The figure below shows that the need for written detail depends on lesson plan's audience.

1. A personal lesson plan is an aid to memory that takes into consideration one's expertise (teaching and subject area knowledge, skills, and experience). It's often quite short—sometimes just a brief list of topics to be covered or ideas to be discussed. (For example, a teacher quite familiar with a geoboard might do a lesson on the geometric shapes that have specific names that one can make on a geoboard. Examples include triangle, and right triangles. Can one make an isosceles triage or an equilateral triangle?) One can make a square

and a rectangle. Can one make a circle? The teacher's lesson plan might say, "Explore use of a geoboard to make geometric shapes. Which of the shapes have names that are familiar to students the teacher is teaching? Do some "how many" activities in which students to find and count how many of a particular geometric figure can be made on a geoboard. For example, how many squares of area 1, 2, 3, 4, 5, etc.?"

2. A collegial lesson plan is designed for a limited, special audience such as your colleagues, a substitute teacher, or a supervisor such as a principal. It contains more detail than the first category. It is designed to communicate with people who are familiar with the school and curriculum of the lesson plan writer.

3. A (high quality) publishable lesson plan is designed for publication and for use by a wide, diverse audience. It contain still more detail than the second category. It is designed to communicate with people who have no specific knowledge of the lesson plan writer's school, school district, and state. It is especially useful to preservice teachers, to substitute teachers in unfamiliar situations, and to workshop presenters seeking to elicit in-depth discussion.

Reading and Video-Viewing Assignment

1. Moursund, D.G. (2008). Good math lesson plans. Retrieved 5/21/2011: http://iae-pedia.org/Good_Math_Lesson_Plans.

Graded Assignment

Assignment # 8 (100 points). All assignments are to be submitted as attachments to an email message sent to the course instructor. The assignment should be done using Microsoft Word or some other commonly used word processing system that includes a spelling check and sufficient desktop publishing capabilities to do an acceptable level of desktop publication of the document. It is assumed that submitted documents will have been carefully spell checked and be of a writing quality that one expects of graduate students.

This assignment is to write one document that contains (and integrates as seems appropriate) the key ideas from the reading and video assignment with the work you do on the following project.

1. Create a Level 3 math lesson plan on a topic that supports the math maturity development ideas in this course and is not a lesson that you have taught before. Before selecting a topic, first read the Unit # 9 material and assignment. Please do not overlap this Unit #8 assignment with the Unit # 9 assignment.

   Test your Level 3 lesson plan with your students, Write up brief "field" notes on what works, what does not work, and so on. Then revise, polish, and submit the lesson plan along with your field notes.

In addition, note that Assignment #10 (the last assignment in this course) is partly based on the assumption that you are making additions to you personal professional Electronic Digital Filing Cabinet on a regular basis. Please **do not** provide comments on that activity as you turn in each assignment. Instead, at the time you submit an assignment, think about the assigned and other readings and video viewings you have done since submitting the previous assignment. Based on this, consider making one or more additions to your EDFC. Remember that entries in your EDFC should be “stuff” for which you have some personal ownership and interest. If can be materials specifically useful to your professional development, materials that you might want to share with other educators, and materials you want to use in your teaching.

Unit # 9: Math Project and Problem-Based Learning

> "Give a man a fish and you feed him for a day. Teach a man to fish and you feed him for a lifetime." (Chinese proverb.)
>
> "Tell me, and I will forget. Show me, and I may remember. Involve me, and I will understand." (Confucius; Chinese thinker and social philosopher; 551 BC – 479 BC.)

Quick Overview of This Unit

Project-based and problem-based learning are used in both precollege education and higher education. Both can be used in math education.

Project-Based Learning is an individual or group activity that goes on over a period of time, resulting in a product, presentation, or performance. The project typically has a time line and milestones, and other aspects of formative evaluation as the project proceeds.

Quite likely you are familiar with the idea of process writing. Think of the task of writing a document as being a project to be carried out. A six-step version of process writing consists of:

1. Brainstorming to come up with ideas relevant to the proposed content.
2. Organizing the brainstormed ideas. Probably you were taught the strategy, "Make an outline."
3. Developing a draft of the document. While doing this, you are continually involved in doing formative self-assessment.
4. Obtaining feedback. It is very important to learn to provide feedback to yourself. Feedback can also come from peers, teachers, and so on.
5. Revising, which may involve going back to earlier steps. Indeed, at any step in this writing process, it may prove necessary or desirable to go back to an earlier step. This is one of the reasons why it is quite useful to learn to compose at a computer.
6. Publishing. This final step can involve cleaning up the spelling, grammar, and sentence structure. In a computer environment, it also involves doing the design and layout to product a professionally looking document.

Similar steps pertain to doing any project. In simple terms, "revise, revise, revise" in a major component of doing a good project.

A **problem**-based learning problem is a problem that might well be assigned to each individual in the class. The problem might be a "case study" as in a business problem situation, or a design problem as in architecture. Typically the problem is quite challenging and requires a student to draw on a great deal of his or her previous knowledge and experience. Such problems tend to be relatively open ended with a number of possible different solutions.

In math problem-based learning, these characteristics distinguish such a problem from the typical "exercise" at the end of a unit or end of a chapter in a math text. (Of course, some math texts contain such challenging problems.) See, for example, http://www2.edc.org/MathProblems/. Quoting from this site:

> **Combine understanding with skill!** Problems "with a Point" help students in grades 6-12 learn new mathematical ideas by building on old ones. Each problem or sequence focuses on one mathematical idea and also connects that idea with others. Varying in difficulty and approaches, these problems are useful for teachers, students, parents, math clubs, and home-schoolers. Problems are classified by topic, time required, suggested technology, required mathematical background, and habits of mind that students develop or use as they work.

Here is an example of a problem from this site:

Mystery Addition

Correct use of units is important in calculations. Sometimes, two plus two does not equal four, as in “two feet plus two inches equals....” Sometimes, 10 + 120 = 1/5, as in “10 min+120 sec = 1/5 hour.” Sometimes... why don’t you see for yourself?

Fill the blank spaces with units (inches, dimes, hours, etc.) in a way that makes the addition examples correct. Try to invent more than one set of answers for each question.

Example: 1... + 1... = 3...
Possible solutions:
1 pint + 1 cup = 3 cups or
1 quarter + 1 eighth = 3 eighths or
1 dime + 1 nickel = 3 nickels

Problems to assign to students:

1. 3... + 2... = 32...
2. 3... + 2... = 23...
3. 3... + 2... = 17...
4. 3... + 2... = 35...
5. 3... + 2... = 182...
6. 3... + 2... = 123...

Note that this example also lends itself to having students create problems they can use to challenge their fellow students. Also, a student does not know in advance whether a problem has zero solutions, exactly one solution, or more than one solution.

For another example suitable for use by a wide range of students, see the “Four Fours Problem” at http://www.wheels.org/math/44s.html.

Reading and Video-Viewing Assignment

1. Moursund, D.G. (2008). Project-based learning. Retrieved 5/21/2011 from http://iae-pedia.org/Project-Based_Learning.
2. Moursund, D.G. (2008). Math project-based learning. Retrieved 5/21/2011 from http://iae-pedia.org/Math_Project-based_Learning.

3. Moursund, D.G. (2008). Good project-based learning lesson plans. Retrieved 5/21/2011from http://iae-pedia.org/Good_PBL_Lesson_Plans.

Graded Assignment

Assignment # 9 (100 points). All assignments are to be submitted as attachments to an email message sent to the course instructor. The assignment should be done using Microsoft Word or some other commonly used word processing system that includes a spelling check and sufficient desktop publishing capabilities to do an acceptable level of desktop publication of the document. It is assumed that submitted documents will have been carefully spell checked and be of a writing quality that one expects of graduate students.

This assignment is to write one document that contains (and integrates as seems appropriate) the key ideas from the reading and video assignment. The specific assignment is:

1. Develop a math project-based learning lesson that you feel is appropriate for use with your students. Develop this lesson plan in sufficient detail so that it could be used by a substitute teacher. Then use the lesson plan with your students and report on the results. Discuss what worked, what did not work, what you learned, and so on.
2. Locate one or more math problems that you feel are appropriate to use in a problem-based learning lesson with your students. Develop this lesson plan in sufficient detail so that it could be used by a substitute teacher. Then use the lesson plan with your students and report on the results. Discuss what worked, what did not work, what you learned, and so on.

In addition, note that Assignment #10 (the last assignment in this course) is partly based on the assumption that you are making additions to you personal professional Electronic Digital Filing Cabinet on a regular basis. Please **do not** provide comments on that activity as you turn in each assignment. Instead, at the time you submit an assignment, think about the assigned and other readings and video viewings you have done since submitting the previous assignment. Based on this, consider making one or more additions to your EDFC. Remember that entries in your EDFC should be "stuff" for which you have some personal ownership and interest. If can be materials specifically useful to your professional development, materials that you might want to share with other educators, and materials you want to use in your teaching.

Unit # 10. Wrap-up

> "Education is what remains after one has forgotten what one has learned in school. "(Albert Einstein; German-born theoretical physicist and 1921 Nobel Prize winner; 1879–1955.)
>
> "Ability will never catch up with the demand for it." (Confucius; Chinese thinker and social philosopher, whose teachings and philosophy have deeply influenced Chinese, Korean, Japanese, Taiwanese and Vietnamese thought and life; 551 BC–479 BC.)

Short Summary of Big Ideas in the Course

Quoting from Unit 2 of this Syllabus:

> Looking specifically within the discipline of math, a mathematically mature adult has the math knowledge, skills, attitudes, perseverance, and experience to be a responsible adult citizen in dealing with the types of math-related situations, problems, and tasks that occur in the societies and cultures in which he or she lives. In addition, a mathematically mature adult knows when to ask for help, both from other people and from tools. There is no shame in having your income taxes prepared by an expert or for you to make use of income tax preparation software. There is no shame in making use of a financial planning and investment expert.

The overall goal of the course in to help increase the math maturity of precollege students, especially those at the K-8 levels. As opposed to other approaches to improving math education, his approach places increased emphasis on developing student understanding and habits of mind that will persist long after memorized details have been forgotten.

Unit 2 of the Syllabus contains a 10-item list of some of measures of a student's increasing math maturity. This is not intended to be a comprehensive list. Rather, it is intended to give a math teacher some math maturity insights that can help in shaping his or her math teaching.

These math maturity insights can be implicitly and/or explicitly integrated into math lessons at all grade levels. For teachers of math, these insights can become part of their math pedagogical knowledge and skills. For the convenience of readers, a slightly less wordy version of the 10-item list from Unit 2 is given here:

1. Read, write, speak, and listen using standard notation, vocabulary, and acceptable style.
2. Learn to learn math; complete the significant shift from learning by memorization to learning through understanding. Take an increasing level of responsibility for one's own learning.
3. Transfer one's math knowledge and skills into math related areas and problems in disciplines outside of mathematics.
4. Work with, think about, and think using abstract math ideas.
5. Develop mathematical intuition. Learn to draw on one's accumulated subconscious math knowledge and insights.

6. Move back and forth between the visual (e.g., graphs, geometric representations) and the analytical e.g., (equations, functions).
7. Recognize a valid mathematical or logical argument or proof, and detect 'sloppy' thinking. Provide solid evidence (informal and formal arguments and proofs) of the correctness of one's math work.
8. Recognize mathematical patterns in problems and problem situations that can be represented and analyzed using math.
9. Represent (model) verbal and written word (story) problem situations as mathematical problems.
10. Develop the confidence, knowledge, skills, and persistence (delayed gratification) to effectively draw upon one's math knowledge and skills to address novel (not previously encountered) math-related problems.

You probably remember the following quote: "As the twig is bent so grows the tree" or perhaps the variation, "As the twig is bent, so grows the tree." Bending a child's informal and formal math education toward increasing math maturity can begin well before students begin their formal schooling.

Reading and Video-Viewing Assignment

There are no assigned readings videos for this unit.

Graded Assignment

Assignment # 10 (100 points). All assignments are to be submitted as attachments to an email message sent to the course instructor. The assignment should be done using Microsoft Word or some other commonly used word processing system that includes a spelling check and sufficient desktop publishing capabilities to do an acceptable level of desktop publication of the document. It is assumed that submitted documents will have been carefully spell checked and be of a writing quality that one expects of graduate students.

1. Write about what you have learned that is useful to you and to your students.
2. Write about what was covered but that you feel was less useful or not useful to you and to your students.
3. Make some specific suggestions on how to improve the course. Keep in mind that the ultimate goal of the course is to improve the quality of education that your students are obtaining.
4. See the list of 10 components of Math Maturity in given earlier in this Unit. This list has helped guide the development of this course. (Remember, these are not all of the components of Math Maturity.) Order the list of items from 1 to 10, with 1 being the most important, on the basis of your opinion of the most important ideas to stress in working with K-8 students to help increase their levels of math maturity. Include comments about your choices if you want to.

A score of 50 points will be given for a good faith, well thought, and well written effort on parts 1-4 of this assignment. "Brown nosing" or being quite critical will not affect your grade on this assignment.

5. (50 points). Provide information about the regular growth of and use of your personal professional Electronic Digital Filing Cabinet during this course. Entries in your EDFC are "stuff" for which you have some personal ownership and interest. If can be materials specifically useful to your professional development, materials that you might want to share with other educators, and materials you want to use in your teaching.

   What have your added to your EDFC during this course? How have you made use of your EDFC during this course? Discuss the nature and extent of sharing of your materials that you have done during this course. Suggest ways to make the EDFG component of the course more useful or effective.

   As in the EDFC Assignment # 3, your course instructor needs to be able to review your EDFC additions that are relevant to and/or inspired by this course. A score of 50 points will be given for a good faith, well thought, and well written effort on part 4 of this assignment

Appendix: Chapter 2 of Book by Moursund and Albrecht

The entire book is available online at:

Moursund, David and Albrecht, Robert (2011). Using math games and word problems to increase math maturity. Eugene, OR: Information Age Education. Download the PDF file from http://i-a-e.org/downloads/doc_download/211-using-math-games-and-word-problems-to-increase-the-math-maturity-of-k-8-students.html. Download the Microsoft Word file from http://i-a-e.org/downloads/doc_download/210-using-math-games-and-word-problems-to-increase-the-math-maturity-of-k-8-students.html.

Here is Chapter 2.

> "God created the natural numbers. All the rest [of mathematics] is the work of man." (Leopold Kronecker; German mathematician; 1823-1891.)
>
> To understand mathematics means to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems. (George Polya; Hungarian mathematician; 1887–1985.)

Math has long been a required part of the school curriculum. This is because some math knowledge, skills, and ways of thinking are deemed important for all students.

We know that math is quite useful in helping to represent and solve problems in many different academic situations as well as in many situations people encounter at home, at work, and at play. We know that the overall field of mathematics is very large and it is still growing.

We also know that students taking math courses vary widely in:

1. How much math they have "covered" (had a reasonable opportunity to learn) in the math courses and informal math learning opportunities they have had.
2. How well they learn, understand, communicate in, and think using the oral and written language of math.
3. How well they can apply their knowledge and skills in a variety of math-related problem-solving situations.
4. How well and how long they retain the math they have learned.

These four topics all relate to math maturity. A child's progress in each of these four areas is an indicator of the child's growing level of math maturity.

This chapter provides some background information about math maturity. An understanding of this chapter is fundamental to understanding and making effective use of the math-related games and math-related word problems presented in later chapters.

Introduction to Brain Science

The brain of a newborn healthy child has a number of built-in capabilities and potentials. The term *plasticity* is often used to describe a brain's ability to change over time. Changes are produced through informal and formal education, training, and experiences, as well as through reaction to disease, injuries, and drugs. Poor nutrition can severely damage a brain.

It takes about 25 years or so for a person's brain to gain its full physical maturity. Even after reaching physical maturity, a healthy brain maintains considerable plasticity and ability to learn.

The brain of a healthy newborn child is naturally curious and creative. It has a great ability to learn. When you learn, the learning takes place in your brain and the rest of your body. Learning in a biochemical process, with changes occurring at a cellular level.

Through study and practice, considerable learning can be incorporated into your brain/body at a subconscious or reflex level. You are familiar with this in situations such as keyboarding using a computer keyboard, playing a musical instrument, dancing and sports, playing action video games, tying your shoelaces, and so on.

But, have you also thought about this in terms of learning oral communication? Your brain has learned to hear sequences of word sounds, and automatically translate them into meaning. Through instruction and practice in reading, your brain has learned to translate "squiggles" (writing) on a piece of paper into meaningful ideas. In addition, your brain has learned to automate the speaking and writing components of communication.

Now, consider the same general ideas, but apply them to communication in the language of mathematics. Very young infants have a little bit of innate ability to recognize small quantities, such as 1, 2, and 3. Recent research suggests we have some innate ability to deal with simple fractions such as 1/2 or 1/3. However, it takes many years of informal and formal education, training, and practice to understand and effectively deal with the language of mathematics at a level deemed appropriate in our current society.

These same ideas also hold for learning in any other discipline of study and practice. Your brain has tremendous versatility, plasticity, and ability to learn. It also has innate creativity. Probably you are familiar with the French mathematician René Descartes' (1596–1650) statement, "I think, therefore I am." A more modern version of this statement might be: "I think consciously and creatively, therefore I am."

Through education, training, and practice your brain can develop considerable math-related automaticity in oral and written communication in the language of math, and thinking in the language of math. The information to be learned can come from:

- sources internal to your body (including from your brain);
- sources outside your body via your senses.

The book you are currently reading is an example of an external source of information. As you think about what you are reading, you are drawing on information freshly stored in your brain as well as information you have stored in your brain over past years.

As an example of learning from internal sources of information, suppose that you are in a sensory deprivation tank. While in the tank, you can still think about information stored in your brain, you can learn by combining this information in new ways, and you can pose and solve problems. For another example, we now know that a lot of learning and unlearning occurs at a subconscious level while one is asleep. We also know that one's subconscious can work on a problem that has been brought into one's brain by previous learning and thinking efforts.

Once data comes into your brain, much of it is processed at a subconscious level. This processing builds on your current knowledge and skills. As an example, consider all of the data bombarding your senses as you walk along a busy street or in the woods. Unless you are paying very careful attention, most of the data coming in through your senses is ignored (filtered out) without ever impinging on your consciousness.

One of the goals in schooling is to help students get better at focusing their attention on the content to be learned. One of the characteristics of a good teacher is being able to attract and hold students' attention. One of the characteristics of a good student is being able to focus attention on what is being taught. As a child's brain grows and matures, it increases in its capabilities to focus attention.

One of the characteristics of increasing "learning maturity" is increasing ability to focus one's attention on what is to be learned. Quoting Michael Posner, a world-class researcher on the topic of attention (Fermandez, 2008):

> … there is not one single "attention", but three separate functions of attention with three separate underlying brain networks: alerting, orienting, and executive attention.
>
> 1) Alerting: helps us maintain an Alert State. [To read and understand a sentence, you must be in an alert state.]
>
> 2) Orienting: focuses our senses on the information we want. For example, you are now listening to my voice. [You hear Mike Posner's voice through his writings.]
>
> 3) Executive Attention: regulates a variety of networks, such as emotional responses and sensory information. This is critical for most other skills, and clearly correlated with academic performance. … [This is why teachers spend so much effort trying to get students to pay attention.]
>
> The development of executive attention can be easily observed both by questionnaire and cognitive tasks after about age 3–4, when parents can identify the ability of their children to regulate their emotions and control their behavior in accord with social demands.

Math Maturity Food for Thought. You have undoubtedly heard of Attention Deficit Disorder (ADD) and Attention Deficit Hyperactive Disorder (ADHD), so you know that people vary considerably in how well they can focus their attention. The discussion given above suggests that our sensory systems and brain are designed to not pay conscious attention to most of the data coming in through our senses.

You have also heard of the idea of multitasking. In multitasking, one pays attention to two or more tasks at the same time. Analyze your strengths, weaknesses, and personal experiences in multitasking. Relate your insights to your own learning experiences and to your experiences as a teacher.

In summary, this section presents a somewhat simplistic model of learning that consists of three key ideas:

1. All of the learning you do occurs inside your brain and the rest of your body. Learning involves biochemical changes at the cellular level in your brain and in the rest of your body.
2. The data that is processed to produce learning can come from internal and external sources. In both cases, the learning that occurs is based on (constructed upon) what has been learned in the past. That is the essence of the learning theory called constructivism. For some information about constructivism in math education, see the Math Forum site http://mathforum.org/mathed/constructivism.html.
3. Paying conscious, alert attention to the topics and ideas you are trying to learn, reflecting on them, and doing metacognition (thinking about your own thinking) on them can help direct the subconscious learning process. (Note, however, in kinesthetic learning of sports, dancing, and so on, conscious attention and careful thinking can often get in the way of developing the mind/body subconscious interactive coordination and automaticity that is needed to attain a high level of performance. The goal is a type of automatic, subconscious type of performance.)

Learning Without and With Understanding

You know about the idea of rote memory with little or no understanding. Such learning is a major component of learning by very young children. You also know the importance of learning with deep and long lasting understanding. As a child gains increased cognitive capabilities, the child gradually moves from rote memory learning with very little understanding toward learning with considerable understanding.

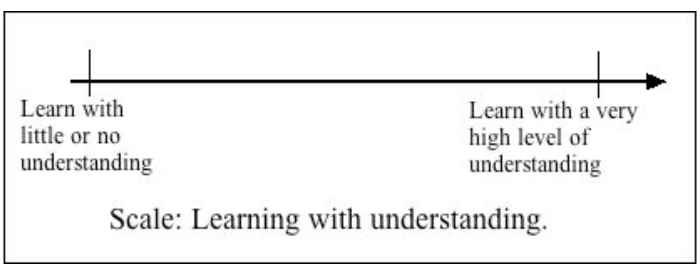


Figure 2.1 An increasing level of maturity in learning.

This progress toward learning with understanding can be thought of as progress toward greater cognitive maturity. It comes about through a combination of nature (genetic dispositions; your genetic blueprint) and nurture. Nurture includes things such as food, clothing, shelter, health care, protection against various dangers, informal education, formal education, and so on. Nurture also includes the loving and nurturing care that a parent and others can give to a child.

As adults, we sometimes tend to have little memory of the complex learning tasks we encountered and accomplished as children. Think about a child learning about the ideas of quantity, time, distance, length, area, volume, and so on. These are all very complex ideas.

To take a specific example, consider the challenge a child faces in learning to "tell" and to "understand" time. There is a huge difference between being to read a digital watch and say the numbers that represent the time, and having an understanding of what the numbers mean.

Think about helping a child to understand time. In the "good old days," all we had were analog clocks and watches. The passage of time was indicated by hands moving (rotating) around the center of the clock or watch face. The clock face typically contains only 12 numbers. A minute is the amount of time that it takes for the long skinny hand (the second hand) to make a complete rotation. One can see the second hand moving, and "watch" the passage of a minute.

Math Maturity Food for Thought. In the English language, one can talk about a first object or event, a second object or event, and so on. We talk about second hand goods, second hand information, and second hand smoke. In measuring time, we talk about seconds, minutes, and so on. We talk about the second hand on a clock. Hmm. We have used the word **second** in a variety of ways.

Confusing, right? And, of course, we have words that sound the same but are spelled differently and have different meanings. The words *four, fore,* and *for* are homonyms. A child learning to understand spoken English gradually comes to understand that the exact same sound pattern has different meanings in different contexts. The listener needs to recognize or figure out the context in order to assign correct meaning in homonym situations.

The language of mathematics is designed for very precise communication. However, the language of mathematics draws heavily on natural language. As an example, think about what the words *equal* or *equals* mean in English, and what they mean in math. Give some other examples of the math discipline assigning a quite specific and precise meaning to an English word, where the English word has two or more different meanings. Think about what this situation has to do with math maturity.

Digital timepieces are great things. But, an analog timepiece with rotating hands is a better aid to developing an initial understanding of time than is a digital timepiece. It is more concrete—think in terms of Piaget's concrete operations stage of a person's cognitive development.

Just for the fun of it, let's carry this time-telling learning for understanding a little further. What is a second, or a minute, or an hour? In your mind, compare and contrast this question with the ideas of a day, a month, and a year. A day (rotation of the earth on its axis), month (orbiting of the moon around the earth) and a year (orbiting of the earth around the sun) are words referring to physical, observable events. That is, they have concrete referents. Humans created the abstract concepts we call a second, a minute, and hour. If your head is not yet spinning, think about when and why we have leap years and why there is a difference between a lunar month and a calendar month. Time is a very complex topic!

To summarize this section, we all have some insights into learning with little or no understanding, and learning with understanding. We all have some insights into the range of complexity in the informal and formal education that children are exposed to. Some ideas, such as time, are so complex that they challenge the world's leading physicists. (Perhaps the name Albert Einstein just popped into your consciousness?) Thus, when we say that we want students to learn with understanding, we need to think carefully what level of understanding we are striving for.

Counting and Adding

Children learn counting words as such as one, two, three, and four as they are learning other worlds used in the everyday language environment in which they are raised. By the time children reach kindergarten, most can count the number of objects in a small collection. That is, they can make a one-to-one correspondence between the objects and the counting words, and then say that the number of objects is the last of the counting words that they used in making the correspondence.

This type of counting also allows a young child to add two numbers, such as two and four. Parents typically teach their children this relatively abstract concept. Perhaps a child has learned the following process to add two and four:

> Count (say the words) one, two. That takes care of the "two" part of the two numbers you are adding together. Then say the next counting word (three) as you bend down one finger. Say the next counting word (four) as you bend down a second finger. Say the next counting word (five) as you bend down a third finger. Say the word six as your bend down a fourth finger. You have now bent down four fingers, and this corresponds to the four in the addition problem. You are now all done. The answer is the last counting number (six) you named. (A slightly different approach is to start the finger bending with the one and two, and continue to bend fingers for the three, etc. The result is six bent fingers.)

A variation on this is to make use of small blocks. It is common to use base-10 blocks and other physical math manipulatives in elementary school. Counting on one's figures is such a powerful aid to doing simple arithmetic calculations that it carries on into adulthood for many people.

Think how complex this process is! Moreover, think about the transfer of learning that we expect of a child learning this process. We expect the child to recognize when the counting-to-add process is applicable. We expect the child to learn the algorithm well enough to apply it in situations not previously encountered.

A relatively powerful variation on counting-to-add is called *counting on*. To add three and seven, first select the larger number. Say its name (in this case, seven) and then begin the counting and finger bending process (saying eight, nine, and ten in this case).

Perhaps 2/3 of students discover on their own or are taught this counting on algorithm before they begin the first grade (Bruer, 1999). The correctness of this process depends on the assumption that the sum of two integers is the same regardless of the order of the two numbers. In math language, addition is commutative.

Math Maturity Food for Thought. Check your own level of understanding. Why is seven and three the same as three and seven? What sort of argument might be convincing to you or to a preschool child? Remember, this is a hard question. You "know" that three times five is the same as five times three. But, you also know that three divided by five is not the same as five divided by three. Thus, not all mathematical operations on pairs of integers are commutative.

Can you think of any other math operations that can be carried out on pairs of integers, and that are not commutative? Think about what this situation means in terms of helping a child gain in math maturity.

The Number Line

Quite young children learn counting words such as one, two, three, … along with other frequently used natural language words in the languages they encounter. Children who grow up in bilingual or trilingual home environments readily develop bilingual or trilingual oral communication skills.

Children learn to count before they develop much of an understanding of the number line. In primary school they are apt to see a banner across the front of the room that is centered on 0 and shows a portion of the number line containing some of the positive integers and some of the negative integers.

You (as an adult) know that there are an infinite number of integers—so that the line number line extends "forever" in each direction. You know about the math symbol ∞ (infinity). You may have memorized that "∞ is not a number." Ask yourself: if ∞ is not a number, what is it?

This provides us with another example of learning by rote memory with little understanding, versus learning in a more "mature" fashion with a level of understanding appropriate to your current and possibly future needs.

Let's explore the number line in a little more detail. The number line is a quite complex math idea. For example, it is "obvious" that there are exactly as many positive integers as there are negative integers. (You can form a one-to-one correspondence with +1 corresponding to -1, +2 corresponding to -2, and so on.)

But, consider the correspondence -1 corresponding to +1, -2 corresponding to +4, -3 corresponding to +9, and so on. In this one-to-one correspondence, each negative integer is paired with its square. That is, there is a one-to-one correspondence between the negative integers and the set of positive perfect squares. So, there are as many negative integers as there are positive perfect square integers. This example suggests some of the difficulties in understating ∞.

You know, of course, that the number line contains more than just integers. For example, it contains fractions, such as1/2, 1/3, and 2/3. It contains other rational numbers that are not integers, such as 1 1/4, 5 1/3 and 24 2/3.

And, we could go still deeper. Consider a right triangle with its two shorter sides each being of length 1. The longer side—the hypotenuse— has a length that is not a rational number. The length is the positive square root of 2, and it is an irrational number. The number π (pi) is also an irrational number that you have encountered before. The discovery of rational and irrational numbers represented important events in the history of mathematics.

Math Maturity Food for Thought. Much of the math that students learn in our PreK-8 grades was known by the Greeks more than 2,000 years ago. Thus, for example, the Pythagorean Theorem is named after Pythagoras of Samos. Quoting from the Wikipedia: "Pythagoras the Samian, c. 570-c. 495 BC, was an Ionian Greek philosopher and founder of the religious movement called Pythagoreanism. He is often revered as a great mathematician, mystic and scientist; however some have questioned the scope of his contributions to mathematics and natural philosophy." Thus, when we have students memorize and work to understand the Pythagorean Theorem, we are teaching them a bit of history. However, we are failing to teach any of the important aspects of Pythagoras, his life, and his times. For example, what do you know about Euclid (of Euclidean Geometry fame)? What are your thoughts on this aspect of math education?

In brief summary, the number line is a very complex mathematical object. As one studies and learns about this complex object, there are different levels of learning and understanding. Thus, when math teachers are told to "teach for understanding" and students are expected to "learn with understanding," things are not as simple as they might seem.

This situation leads to questions such as what constitutes an appropriate level of understanding for the number line that should be taught and learned in grade school, in middle school, in high school, or in college?

These types of questions help to complicate the topic of math maturity. Do we say that a student is increasing in math maturity as he or she learns more and more about the number line?

The same types of questions can be asked about any area of math. Certainly one aspect of increasing math maturity is gaining increased knowledge and skill in math. However, our focus on math maturity is not on specific math content. Rather, it is on math-oriented thinking, understanding, remembering, and being able to make effective use of the math that one has had an opportunity to learn.

Math has a High Inherent Level of Abstractness

Much of the power of math lies in its relatively high level of abstractness. Think again about a young child learning the number words one, two, three, etc. The child eventually learns that by saying the words and making a one-to-one correspondence with a set of objects, the final number said is the quantity of objects in the set. That is a major math-learning step.

Later the child encounters the symbols 1, 2, 3, etc. These are shorthand symbols for the words one, two, three, etc. and likely they are learned (memorized) before the child encounters and learns the alphabetic representations of the words one, two, three, etc. Do you find this interesting? We have children learn abstract shorthand representations for the natural language words one, two, three, etc. before we have them learn to read and spell the written forms of these words. That is, very early on in a child's education we move toward the abstractness and power of the language of mathematics.

David Tall is a mathematics education theorist at the University of Warwick in England. Quoting Tall (2000):

> The development of symbol sense throughout the curriculum therefore faces a number of major reconstructions which cause increasing difficulties to more and more students as they are faced with successive new ideas that require new coping mechanisms. For many it leads to the satisfying immediate short-term needs of passing examinations by rote-learning procedures. The students may therefore satisfy the requirements of the current course and the teacher of the course is seen to be successful. However, if the long-term development of rich cognitive units is

> not set in motion, short-term success may only lead to increasing cognitive load and potential long-term failure.

One thing implied here is that, as the symbols and the manipulations become more and more abstract, it becomes more difficult to relate to what is known; the student "learns" with less and less understanding. In many cases, a student would face a daunting task working out a referent that has meaning to the student.

Think about this in terms of a stage theory of cognitive development, such as Piaget's 4-stage model. Newborn children are in the sensorimotor stage. They gradually move into the preoperational stage and then the concrete operations stage. For most children, the concrete operations stage roughly corresponds to the years of elementary school. In the concrete operations stage, the human brain is highly dependent on concrete referents. It struggles with the types of abstractness that are in the math curriculum. And so, many do not gain much understanding as they try to learn how to do arithmetic with fractions.

Piaget's fourth stage is called formal operations. At that level of cognitive development, people are able to deal with a high level of abstractness in math and in other disciplines. High school students in an Advanced Placement history course are expected to function at this level of cognitive development. See http://iae-pedia.org/Digital_Filing_Cabinet/Secondary_School_History.

We will discuss this more in **Chapter 4**. There we argue that the level of abstractness in the math curriculum tends to be quite high relatively to the cognitive development of students. This leads to memorization without much understanding—in order to pass the courses. As David Tall points out in the quote given above, this leads many students to failure (or, dropping out) in their math studies. The Algebra 1 course is a stumbling block for a great many students because they do not understand fractions and calculations involving fractions.

Defining Math Maturity

The Preface and Introduction to this book states:

> A **mathematically mature adult** has the math knowledge, skills, attitudes, perseverance, and experience to be a responsible adult citizen in dealing with the types of math-related situations, problems, and tasks that occur in the societies and cultures in which he or she lives. In addition, a mathematically mature adult knows when and how to ask for and make appropriate use of help from other people, from books, and from tools such as computer systems.

Chapter 1 and earlier parts of this chapter suggest a number of possible components of math maturity. Now, perhaps, you are expecting a definitive definition of what constitutes math maturity and how to measure it. If so, prepare yourself to be disappointed. There is no widely agreed upon definition of math maturity, and there are no assessment instruments that are effective in precisely measuring a person's level of math maturity.

However, there is quite a bit of literature on the topic. A recent Internet search of the expression *"math maturity" OR "mathematical maturity"* produced about 25,000 hits.

Quoting from the Wikipedia (http://en.wikipedia.org/wiki/Mathematical_maturity):

> Mathematical maturity is a loose term used by mathematicians that refers to a mixture of mathematical experience and insight that cannot be directly taught. Instead, it comes from repeated exposure to complex mathematical concepts.
>
> An illustrative example from common experience that may be more familiar to non-mathematicians would be high school geometry proofs. While most competent and interested students can follow a given proof and even determine whether or not it is correct, many still have trouble coming up with a proof. The ill-defined difference between those who can generate proofs and those who cannot is an example of a difference in mathematical maturity, in this case the ability to "see" how to proceed.

The book you are now reading disagrees with the part of the above definition that says, “that cannot be directly taught.” The essence of this book is that informal and formal math education can be designed explicitly to help increase math maturity.

Many discussions of math maturity include a focus on understanding and creating proofs. Consider what this means for younger students. When a student solves a problem and then explains the steps involved in a manner that is convincing to others, the students has (in essence) created a proof. That is, problem solving and making proofs are two sides of the same coin.

George Polya was one of the leading mathematicians of the 20th century, and he wrote extensively about problem solving. *The Goals of Mathematical Education* (Polya, 1969) is a talk that he gave to a group of preservice and inservice math teachers. The talk focused on elementary school math.

> To understand mathematics means to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems. For the higher aims about which I am now talking are some general tactics of problems—to have the right attitude for problems and to be able to attack all kinds of problems, not only very simple problems, which can be solved with the skills of the primary school, but more complicated problems of engineering, physics and so on, which will be further developed in the high school. But the foundations should be started in the primary school. And so I think an essential point in the primary school is to introduce the children to the tactics of problem solving. Not to solve this or that kind of problem, not to make just long divisions or some such thing, but to develop a general attitude for the solution of problems.
>
> …
>
> However, … We wish to develop all the resources of the growing child. And the part that mathematics plays is mostly about thinking. Mathematics is a good school of thinking. But what is thinking? The thinking that you can learn in mathematics is, for instance, to handle abstractions. Mathematics is about numbers. Numbers are an abstraction. When we solve a practical problem, then from this practical problem we must first make an abstract problem. Mathematics applies directly to abstractions. Some mathematics should enable a child at least to handle abstractions, to handle abstract structures.

Polya's comments focus on math problem solving, math thinking, dealing with math abstraction, and other aspects of "doing" math. In essence, the quoted material serves as a definition of math maturity.

Larry Denenberg has a PhD in applied mathematics and is a systems analyst, entrepreneur, and business executive. Quoting from a course syllabus developed by Denenberg (http://www.larry.denenberg.com/math22/LectureA.pdf):

> Thirty percent of mathematical maturity is fearlessness in the face of symbols: the ability to read and understand notation, to introduce clear and useful notation when appropriate (and not otherwise!), and a general facility of expression in the terse—but crisp and exact—language that mathematicians use to communicate ideas. Mathematics, like English, relies on a common understanding of definitions and meanings. But in mathematics definitions and meanings are much more often attached to symbols, not to words, although words are used as well. Furthermore, the definitions are much more precise and unambiguous, and are not nearly as susceptible to modification through usage. You will never see a mathematical discussion without the use of notation!

The remainder of this section provides a list of some generally agreed on components of math maturity. This list, and components that you may want to add to fit your personal insights into math maturity, can help in the design of math curriculum content, instructional processes, and assessment.

The list given below is rather extensive.

1. Communication

Communicate mathematics and math ideas orally and in writing using standard notation, vocabulary, and acceptable style. As noted earlier in this book, an oral communication between two people can be thought of as an exchange of word problems. If the topic is related to and/or involves math in some way, then the two participants are involved in creating, communicating and perhaps solving math word problems.

Here is a more mathematical way of describing an increasing level of the communication component of math maturity. Consider a situation in which a person is using oral language, gestures, written languages, pictures, and diagrams, to communicate a math problem or math related problem to another person. The idea is to communicate the problem carefully and fully, so that the receiver of the communication can then bring his or her math knowledge and skills to bear in attempting to solve the problem. This type of communication requires a high level of precision on the parts of the two participants. An increasing level of math maturity is evidenced by increasing ability to communicate in math and math-related areas.

2. Learn to learn math and help others learn math

Learning math means learning with an appropriate combination of memorization and understanding. Some key ideas include constructivism, metacognition, and reflective thinking. Learning to learn math includes learning to make effective use of the various aids to learning that are available, such as teachers, peers, books, and computers. It also includes leaning to make effective use of one's overall learning knowledge and skills, and one's specific math learning knowledge and skills. An increasing level of math maturity is evidenced by increasing ability to be a self-sufficient intrinsically motivated learner who learns math with understanding.

Increasing math maturity is also evidenced by increasing ability to work with people having varying levels of math knowledge and skills, and to help them learn math.

3. Generalize from a specific example to a broad concept

Mathematicians often start from a specific example of a problem, and go on to represent, define, and solve a broad category of closely related problems. For example, one might start with a concrete example of a problem involving a specific equilateral triangle, and develop results that solve this type of problem for every equilateral triangle. Increasing math maturity includes getting better at identifying a general class of problems from a specific example, in solving the general class of problems, and in making use of a solution to a general class or problems to solve specific instances of the problem.

4. Transfer of learning

One aspect of learning math is to learn a variety of strategies (algorithmic and heuristic) that are useful in attacking a broad range of math problems, and to learn to develop such strategies on one's own. Another aspect of learning math is to learn to think and reason mathematically. Increasing math maturity is evidenced by getting better at transferring or applying one's math knowledge and skills into other areas of math and into math related areas and problems in disciplines outside of mathematics. Progress is shown by increasingly being able to apply one's math knowledge and skills to challenging math-related problems and problem situations that one has not previously encountered.

5. Multiple, varied representations

Children begin their leaning of math well before they reach the "concrete operations" phase of cognitive development. This early math learning is rooted in verbal, tactile, and visual representations of specific concrete objects and events. Increasing math maturity is evidenced in increasing ability to deal with generalizations and less concrete examples. For example, there is a difference between working with three blue toy cars and two red toy cars that one has physically sitting before one's eyes, and doing the same thing with pictures in a book or pictures of cars in one's mind's eye. At a much higher math level, increasing math maturity includes getting better at moving back and forth between the visual (e.g., graphs, geometric representations) and the analytical e.g., (equations, functions) math representations.

6. Math problem solving and proofs

Solving math problems and proving math theorems lie at the very heart of mathematics. Increasing math maturity is evidenced by progress in being able to provide solid evidence (informal and formal arguments and proofs) of the correctness of one's efforts in solving math problems and making proofs. This is a specific type of communication in the language of math. Before students encounter formal math proof processes, math proof is often stated as "Show your work and check your answers." This means to present written explanations and arguments that support and assertion that your work is correct.

7. Math-related word problems

Represent (model) verbal and written problems in any discipline as mathematical problems. Recognize when a word problem might make effective use of math in attempts to solve the problem. Increasing math maturity is evidenced by increasing knowledge and skills in representing word problems using the language of mathematics, solving the resulting math problem, translating the results back into the language and context of the original word problem, and checking for accuracy and mindfulness of the math results in light of the context and meaning of the original word problem.

8. Math is a human endeavor

Math is more than just solving math problems and making math proofs. Our accumulated math knowledge represents considerable human creativity over thousands of years. Math is part of our culture. Math is fun. Math is part of the games we create and play. Math is part of the beauty of our world. (See Stephen Brown, n.d.) Increasing math maturity is evidenced by increased understanding or and participation in these various aspects of the overall discipline of mathematics.

Some related topics include mathematizing, thinking like a mathematician, and (math) problem posing (Stephen Brown 1997). An increasing ability to mathematize (see the math in a problem situation; pose math problems) is an indicator of an increasing level of math maturity.

9. Math content

As you think about the math maturity components listed above, notice that few specific math content topics are mentioned. One needs to know some math in order to be able to demonstrate increasing math maturity. But, increasing levels of math maturity are not dependent on gaining some specified and widely agreed on collection of math content.

An indication of an increasing level of math maturity is a student's active engagement in improving his or her math education and prowess in using this math education. This effort need not be focused on just the curriculum content being offered by one's school. Indeed the ability and interest to explore math-related ideas that happen to seem interesting is a good indicator of an increasing level of math maturity.

Great effort has gone into the development of various curricula and various approaches to learning math. There is a reasonable amount of agreement as to math topics that students should learn something about in elementary school, secondary school, and in undergraduate programs of study. However, it is depth of understanding that is the key idea.

Math is an old and vast discipline. It has great breadth and depth. It is a growing field in its own right, and its uses in other disciplines continue to grow.

The leaders and educators in each discipline design and construct curriculum in the discipline. They want students to gain some of the important knowledge and skills in the discipline and to lay a foundation for future learning within the discipline.

Thus, part of one's increasing math maturity is increasing breadth and depth of mathematical knowledge and skills. However, because of the great breadth and depth of the discipline of math, a mere listing of topics to be studied is a poor approach to improving math maturity. Please reread the words of George Polya quoted earlier in this chapter. In essence, Polya is stressing learning to think logically-mathematically in representing and attacking problems.

> To understand mathematics means to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems. **For the higher aims about which I am now talking are some general tactics of problems—to have the right attitude for problems and to be able to attack all kinds of problems, not only very simple problems, which can be solved with the skills of the primary school, but more**

> **complicated problems of engineering, physics and so on, which will be further developed in the high school.** But the foundations should be started in the primary school. And so I think an essential point in the primary school is to introduce the children to the tactics of problem solving. Not to solve this or that kind of problem, not to make just long divisions or some such thing, but to develop a general attitude for the solution of problems. [Bold added for emphasis.]

In essence, Polya is stressing the need to gain increased skill in representing and solving problems using the math that one has learned. This transfer of learning—moving from one's math content knowledge to being able to effectively make use of the content in representing and solving problems—is a fundamental aspect of increasing math maturity.

10. Mathematical intuition

As one's knowledge of and experience in using math grows, one's math intuition grows. Herbert Simon, a Nobel Prize winning polymath, defined intuition as "frozen analysis." He noted that in any disciplines where one studies and practices extensively, a subconscious type of intuition is developed. This intuition may well be able to quickly detect an error that one has made in math thinking and math problem solving, very quickly decide a way to attack a particular type of problem, or provide a "feeling" for the possible correctness of a conjecture.

As an example, look at a student's statement that 5 + 8 = 40. At a subconscious level your brain might say, "something is wrong." It might next tell you, "the number 40 is way too large." Your experience and math teaching intuition might tell you, "perhaps the student multiplied instead of added." Through grading lots of student papers, you have developed some math intuition that makes you into a faster paper grader.

For a deeper view of math intuition, read Henri Poincaré 1905 paper, Intuition and Logic in Mathematics, available at http://www-history.mcs.st-and.ac.uk/Extras/Poincare_Intuition.html. Quoting the first paragraph:

> It is impossible to study the works of the great mathematicians, or even those of the lesser, without noticing and distinguishing two opposite tendencies, or rather two entirely different kinds of minds. The one sort are above all preoccupied with logic; to read their works, one is tempted to believe they have advanced only step by step, after the manner of a Vauban who pushes on his trenches against the place besieged, leaving nothing to chance. The other sort are guided by intuition and at the first stroke make quick but sometimes precarious conquests, like bold cavalrymen of the advance guard.

We see such intuition in other areas, such as in chess. An accomplished chess player can glance at a board position and have a "feeling" for the threats and opportunities that the position represents.

11. Computers and other math tools

All of the above needs to take into consideration the various tools that have been developed to aid in representing and solving math problems and problems in which math can be a useful aid to their solution.

Calculators and computers are powerful examples of such tools. These tools are useful both in representing and solving math problems and also in learning math. Moreover, computational mathematics is now one of the major subdivisions of the overall field of mathematics. Thus, increasing levels of math maturity are evidenced by increasing knowledge and skills in making effective use of Information and Communication Technology both as an aid to representing and solving math problems and as an aid to learning math. See http://iae-pedia.org/Two_Brains_Are_Better_Than_One.

Math Maturity Food for Thought. To summarize the list given above, a mathematically mature adult has the math and math tools knowledge, understanding, and skills to accomplish the math-related activities in his or her overall set of adult responsibilities and problem-solving challenges.

Here are three questions to ask yourself and/or to discuss with others:

1. How well does your current level of math maturity fit your current needs and your overall aspirations and plans for your future? Think about your understanding of math maturity and arguments to support your answer to the question.
2. How does your current level of math maturity compare with that of your peers and with other people you know?
3. What are some of the things you do that help increase the level of math maturity of students, your children, and other people you interact with?

WordsWorth Plus and Math Maturity

This book contains a number of math-oriented educational games and word problems designed to help students gain in math maturity. One of the goals in this book is to help you—the reader—become better at analyzing a game or word problem from the point of view of its contributions to helping students gain in math maturity.

Keep in mind that math maturity is not something that a student achieves or does not achieve. Rather, math maturity is an open-ended goal. Thus, math maturity is not a unit to be covered in a math course at some specified grade level. Rather, all math instruction should contain aspects designed to help students increase their current level of math maturity.

Now, consider the WordsWorth Plus game and some of its variations presented in Chapter 1. In working with students at some grade level or at some math maturity level, you make a decision to have the students use WordsWorth Plus. Your decision might be based on one or more thoughts such as:

- You feel that the combination of numbers and words in the WordsWorth Plus games will give your students useful insight into relationships among numbers and words, use of tables involving both numbers and words, practice in doing mental arithmetic, and practice in creating patterns of letters and determining if they are words.
- You feel your students need a break from the usual grind of the math curriculum you are using. However, you want this "break" to contribute to your students' overall cognitive growth.
- You want your students to be more actively and creatively engaged in math-related activities and word-related activities in a manner that combines the two.

- You are looking for academically respectable activities that can engage students with a wide range of academic backgrounds and potentials.
- You want to actively participate in game-based action research. See http://www.alliance.brown.edu/pubs/themes_ed/act_**research**.pdf.
- You trust the judgment of the authors David Moursund and Bob Albrecht well enough so that you are willing to experiment with a game that they recommend.

Notice that this type of thinking does not require you try to determine the "one best game" for use with your students. Rather, you think about games that are familiar to you and that you feel with be useful in your teaching. Be open to experimenting with new games that you have been exposed to but have not yet tried out with your students. You think about specific educational goals you want to achieve by using a game. You make a decision to use a particular game based on your insights into your students as well as you understanding of underlying goals such as helping your students gain in math maturity and getting your students more actively engaged in math-related learning activities.

Final Remarks

This chapter gives a variety of approaches to defining math maturity. There is no simple definition of various levels of math maturity and no simple pathway that schools or an individual student can take to achieve specific levels of math maturity.

However, our educational system can help students to understand the concept of increasing levels of math maturity and moving in the direction of increasing math maturity. Our math education system can create curriculum content, instructional processes, and assessment systems that help students increase in math maturity. This book is designed to help in that endeavor.

Activities and Possible Homework Assignments

1. **(For use with students.)** Probably you remember the game, "Mother, may I?" See http://www.ehow.com/how_16079_play-mother-may.html. In this game a leader (typically called mother, but who may be a male or female child in the group) interacts with a group of children who begin arranged in a line facing "mother." Mother selects a child and says something like, "You may take three bunny hops forward." The child responds, "Mother, may I?" Upon receiving permission, the child then takes three bunny hops forward. Of course, large steps, small steps, jumps, and so on can all be used in this game.

   What does this have to do with math and math maturity? Here are a few answers:

   a. Children follow a set of instructions to line up in a certain way—in a straight line.

   b. The leader (mother) singles out a child and provides permission to make a certain movement. The movement is given in terms of a number and a type of movement.

   c. The child must precisely follow the rules of the game by saying, "Mother, may I?"

   d. Upon receiving permission, the child must then follow a set of instructions given in words and a number.

2. **(For use with students.)** Now, consider some variations on the "Mother, may I?" game. Suppose mother has a pair of dice—one red and one green. Mother is the leader of a game in which two other students participate. One student is designed as the "red die" player and the other as the "green die" player. Instead of doing steps of bunny hops, a player moves along a number line, starting at 0 and moving toward larger numbers. Mother rolls the dice. The red die player reads the red die (suppose, for example it is a 4) and says, "Mother, may I move four spaces forward?" Mother responds with "yes" if that is the correct number on the red die, and no if it is not. After red completes his or her move, green follows in a similar manner. The game proceeds for a predetermined number of moves, such as 10 or until a player reaches or exceeds a predetermined target number such as 50

3. **(A possible homework assignment or discussion topic in a course.)** Examine the list of 11 possible components of math maturity given earlier in this chapter. Based on your knowledge of math teaching and learning at the precollege level, select two or three of the components that you feel our math education system does well at, and two or three that it does poorly at. Present arguments to support your choices.

4. **(A possible homework assignment or discussion topic in a course.)** Reflect on your own math maturity. Based on this, do three things:

    a. Suggest one or more components that you feel should be added to the 11-component-list given earlier in this chapter. Justify your recommendation(s).

    b. Select a person (such as a fellow teacher) that you know fairly well. Compare and contrast your level of math maturity and this person's level of math maturity using various items from the 11-item list given earlier in this chapter. Note that this activity can be done in small groups in a class setting.

c. Make some suggestions to yourself on things you could do to increase your current level of math maturity.

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