Mathemagical Black Holes

Bob Albrecht & George Firedrake

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Start with a number from a designated set of numbers. For example, start with a natural number. The natural numbers are 1, 2, 3, 4, 5, and so on forever. Natural numbers are also called counting numbers or positive integers. Or start with a whole number. The whole numbers are 0, 1, 2, 3, 4, 5, and so on forever.

Apply a process (a sequence of mathematical operations, a step-by-step procedure) to the starting number and get a new number. If the new number is the same as the starting number, then the starting number is a mathemagical black hole using that process.

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Start with a number from a designated set of numbers. For example, start with a natural number. The natural numbers are 1, 2, 3, 4, 5, and so on forever. Natural numbers are also called counting numbers or positive integers. Or start with a whole number. The whole numbers are 0, 1, 2, 3, 4, 5, and so on forever.

Natural numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, ...
Whole numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ...

Apply a process (a sequence of mathematical operations, a step-by-step procedure) to the starting number and get a new number. If the new number is the same as the starting number, then the starting number is a mathemagical black hole using that process. Apply the process again and get the same number. Black hole.

Add Zero Black Hole. Start with any whole number. Add zero (0) to the starting number. The new number is equal to the starting number. Using the add zero process, every whole number is a black hole. Abracadabra!

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 + 0 = 0</td>
</tr>
<tr>
<td>1 + 0 = 1</td>
</tr>
<tr>
<td>2 + 0 = 2</td>
</tr>
<tr>
<td>3 + 0 = 3</td>
</tr>
<tr>
<td>4 + 0 = 4</td>
</tr>
<tr>
<td>0 + 1 = 1</td>
</tr>
<tr>
<td>0 + 2 = 2</td>
</tr>
<tr>
<td>0 + 3 = 3</td>
</tr>
<tr>
<td>0 + 4 = 4</td>
</tr>
</tbody>
</table>
For any starting number,

\[ \text{starting number} + 0 = \text{starting number} \]

and

\[ 0 + \text{starting number} = \text{starting number} \]

Using the add zero process,
every whole number is a black hole.

**Your Turn** Complete this table before you peek at our answers. The completed table will show that, for any starting number 5 to 9, the starting number plus zero (0) is equal to the starting number, and zero (0) plus the starting number is equal to the starting number.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>0</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

**Answers**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>0</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

The Add Zero Black Hole is a black hole of the simple kind. It works for any whole number.

- Start with any whole number. Add zero (0) to the starting number. The new number is equal to the starting number.
- Using the add zero process, every whole number is a black hole. Alakazam!

Zero (0) is the additive identity. If \( a \) is a whole number, then

\[ a + 0 = a \text{ and } 0 + a = a. \]

Using the add zero process,
every whole number is a black hole.
**Subtract Zero Black Hole.** Start with any whole number. Subtract zero (0) from the starting number. The new number is equal to the starting number. Using the Subtract Zero process, every whole number is a black hole. Presto!

**Example.** Start with 7. Subtract 0 from the starting number \((7 - 0 = 7)\). The new number is 7, the same as the starting number. Using the Subtract 0 process, every whole number is a black hole.

\[
\text{If } a \text{ is a whole number, then } a - 0 = a.
\]

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 0 = 0</td>
</tr>
<tr>
<td>1 – 0 = 1</td>
</tr>
<tr>
<td>2 – 0 = 2</td>
</tr>
<tr>
<td>3 – 0 = 3</td>
</tr>
<tr>
<td>4 – 0 = 4</td>
</tr>
</tbody>
</table>

**Your Turn** Complete this table before you peek at our answers. The completed table will show that, for any starting number 5 to 9, the starting number minus zero (0) is equal to the starting number.

\[
5 – 0 = ____  \quad 6 – 0 = ____  \quad 7 – 0 = ____  \quad 8 – 0 = ____  \quad 9 – 0 = ____.\]

**Answers**

\[
5 – 0 = 5 \quad 6 – 0 = 6 \quad 7 – 0 = 7 \quad 8 – 0 = 8 \quad 9 – 0 = 9\]

The Subtract Zero Black Hole is a black hole of the simple kind. It works for any whole number.

- Start with any whole number. Subtract zero (0) from the starting number. The new number is equal to the starting number.
- Using the subtract zero process, every whole number is a black hole. Alakazam!

\[
\text{Using the subtract zero process,} \\
\text{every whole number is a black hole.}
\]
Multiply by One Black Hole. Start with any whole number. Multiply the starting number by one (1). The new number is the same as the starting number. Using the multiply by 1 process, every whole number is a black hole.

Example. Start with 6. Multiply the starting number by 1 (6 × 1 = 6 or 1 × 6 = 6). The new number is 6, the same as the starting number. Using the multiply by 1 process, every whole number is a black hole.

One (1) is the multiplicative identity. If \( a \) is a whole number, then \( a \times 1 = a \) and \( 1 \times a = a \).

<table>
<thead>
<tr>
<th>Examples</th>
<th>0 × 1 = 0</th>
<th>1 × 1 = 1</th>
<th>2 × 1 = 2</th>
<th>3 × 1 = 3</th>
<th>4 × 1 = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 × 0 = 0</td>
<td>1 × 1 = 1</td>
<td>1 × 2 = 2</td>
<td>1 × 3 = 3</td>
<td>1 × 4 = 4</td>
<td></td>
</tr>
</tbody>
</table>

Your Turn Complete this table before you peek at our answers. It shows that 1 times any starting number from 5 to 9 is equal to the starting number.

<table>
<thead>
<tr>
<th></th>
<th>5 × 1 =</th>
<th>6 × 1 =</th>
<th>7 × 1 =</th>
<th>8 × 1 =</th>
<th>9 × 1 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 × 5 =</td>
<td>___</td>
<td>1 × 6 =</td>
<td>1 × 7 =</td>
<td>1 × 8 =</td>
<td>1 × 9 =</td>
</tr>
</tbody>
</table>

Answers

<table>
<thead>
<tr>
<th></th>
<th>5 × 1 = 5</th>
<th>6 × 1 = 6</th>
<th>7 × 1 = 7</th>
<th>8 × 1 = 8</th>
<th>9 × 1 = 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 × 5 =</td>
<td>5</td>
<td>1 × 6 = 6</td>
<td>1 × 7 = 7</td>
<td>1 × 8 = 8</td>
<td>1 × 9 = 9</td>
</tr>
</tbody>
</table>

The Multiply by One Black Hole is a black hole of the simple kind.

- Start with any whole number. Multiply the starting number by 1. The new number is the same as the starting number.
- Using the multiply by 1 process, every whole number is a black hole.

Using the multiply by 1 process, every whole number is a black hole.
**Divide by One Black Hole.** Start with any whole number. Divide the starting number by one (1). The new number is the same as the starting number. Using the divide by 1 process, every whole number is a black hole.

We use the right-leaning slash (/) as our division symbol. If \( a \) and \( b \) are whole numbers, then \( a \div b \) means “divide \( a \) by \( b \).”

**Example.** Start with 8. Divide the starting number by 1 (8 \( \div \) 1 = 8). The new number is 8, the same as the starting number. Using the divide by 1 process, every whole number is a black hole.

If \( a \) is a whole number, then \( a \div 1 = a \).

| Examples     | \( 0 \div 1 = 0 \) | \( 1 \div 1 = 1 \) | \( 2 \div 1 = 2 \) | \( 3 \div 1 = 3 \) | \( 4 \div 1 = 4 \) |

**Your Turn** Complete this table before you peek at our answers. The completed table shows that any starting number 5 to 9 divided by 1 is equal to the starting number.

| 5 / 1 = ___ | 6 / 1 = ___ | 7 / 1 = ___ | 8 / 1 = ___ | 9 / 1 = ___ |

| Answers     | 5 / 1 = 5 | 6 / 1 = 6 | 7 / 1 = 7 | 8 / 1 = 8 | 9 / 1 = 9 |

The Divide by One Black Hole is a black hole of the simple kind.

- Start with any whole number. Divide the starting number by 1. The new number is the same as the starting number.
- Using the divide by 1 process, every whole number is a black hole.
Ahoy Teachers and Tutors. The Add Zero, Subtract Zero, Multiply by One, and Divide by One black holes reinforce important properties of numbers. Use black holes to help students learn mathemagical stuff.

If \( a \) is a whole number, then \( a + 0 = a \) and \( 0 + a = a \).

If \( a \) is a whole number, then \( a - 0 = a \).

If \( a \) is a whole number, then \( a \times 1 = a \) and \( 1 \times a = a \).

If \( a \) is a whole number, then \( a / 1 = a \).

Double and Divide by 2 Black Hole. Start with any whole number. Double the starting number and then divide that result by 2. Using the double and divide by 2 process, every whole number is a black hole.

Example. Start with 4. Double the starting number (\( 2 \times 4 = 8 \)) and then divide that result by 2 (\( 8 / 2 = 4 \)). The new number is 4, the same as the starting number. Black hole.

<table>
<thead>
<tr>
<th>Start with</th>
<th>Double it</th>
<th>Divide by 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

Examples Using Base 10 Unit Cubes

<table>
<thead>
<tr>
<th>Start with</th>
<th>Double it</th>
<th>Divide by 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>■</td>
<td>■■</td>
<td>■</td>
</tr>
<tr>
<td>■■</td>
<td>■■■■■■</td>
<td>■■</td>
</tr>
<tr>
<td>■■■</td>
<td>■■■■■■■■■■</td>
<td>■■■</td>
</tr>
<tr>
<td>■■■■</td>
<td>■■■■■■■■■■■</td>
<td>■■■■</td>
</tr>
</tbody>
</table>
Your Turn Complete this table before you peek at our Answers. The completed table shows that for any starting number 5 to 9, doubling the starting number and then dividing by 2 results in the starting number.

<table>
<thead>
<tr>
<th>Start with</th>
<th>Double it</th>
<th>Divide by 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>_____</td>
<td>_____</td>
</tr>
<tr>
<td>6</td>
<td>_____</td>
<td>_____</td>
</tr>
<tr>
<td>7</td>
<td>_____</td>
<td>_____</td>
</tr>
<tr>
<td>8</td>
<td>_____</td>
<td>_____</td>
</tr>
<tr>
<td>9</td>
<td>_____</td>
<td>_____</td>
</tr>
</tbody>
</table>

Answers

<table>
<thead>
<tr>
<th>Start with</th>
<th>Double it</th>
<th>Divide by 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>18</td>
<td>9</td>
</tr>
</tbody>
</table>

The Double and Divide by 2 Black Hole is a black hole of the simple kind.

- Start with any whole number. Double the starting number and then divide that result by 2.
- Using the double and divide by 2 process, every whole number is a black hole.

Using the **double and divide by 2** process, every whole number is a black hole.

**Question.** Is tripling a number and then dividing the result by 3 a black hole process?

**Question.** Is multiplying a number by \(n\) and then dividing the product by \(n\) a black hole process? What whole number would you NOT choose as the value of \(n\)?
More Interesting Black Holes

Another type of black hole is more interesting. Start with any number and apply a process one or more times until you arrive at a black hole. Same black hole for any starting number! Hocus pocus, here we go with easy examples.

**Subtract a Number from Itself Black Hole.** Start with any whole number. Subtract the number from itself. The result is zero (0). Only one application of the subtract a number from itself process is required to reach black hole 0. Subtract zero from zero and get zero (0 – 0 = 0). Start with any whole number, subtract it from itself, and end up in black hole 0.

**Example.** Start with 3. Subtract the starting number from itself (3 – 3 = 0). Using this process, every starting number ends up in black hole 0.

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 0 = 0</td>
</tr>
</tbody>
</table>

**Your Turn** Complete this table. The completed table shows that for any starting number 5 to 9, subtracting the starting number from itself results in black hole 0.

| 5 – 5 = ___ | 6 – 6 = ___ | 7 – 7 = ___ | 8 – 8 = ___ | 9 – 9 = ___ |

<table>
<thead>
<tr>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 – 5 = 0</td>
</tr>
</tbody>
</table>

The Subtract a Number from Itself Black Hole is a black hole of the simple kind.

- Start with any whole number. Subtract the number from itself. The result is 0. Only one application of the subtract a number from itself process is required to reach black hole 0.
- For any starting number, starting number – starting number = 0.

Using the **subtract a number from itself** process, every whole number falls into **black hole 0**.

If $a$ is a whole number, then $a – a = 0$. 
Divide a Number by Itself Black Hole. Start with any natural number. Divide the number by itself. The result is black hole 1.

This process doesn’t work for 0. Dividing by 0 is a no no.

Example. Start with 9. Divide the starting number by itself \((9 / 9 = 1)\). Using the divide a number by itself process, every starting number ends up in black hole 1.

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 / 0 no no</td>
</tr>
</tbody>
</table>

Your Turn Complete this table. The completed table shows that dividing any starting number from 5 to 9 by itself results in black hole 1.

\[
\begin{array}{ccccc}
5 / 5 = \_
\end{array}
\]
\[
\begin{array}{ccccc}
6 / 6 = \\
7 / 7 = \\
8 / 8 = \\
9 / 9 = \\
\end{array}
\]

**Answers**

\[
\begin{array}{ccccc}
5 / 5 = 1 & 6 / 6 = 1 & 7 / 7 = 1 & 8 / 8 = 1 & 9 / 9 = 1
\end{array}
\]

The Divide a Number by Itself Black Hole is a black hole of the simple kind.

- Start with any natural number. Divide the number by itself. The result is black hole 1.
- This process doesn’t work for 0. Division by 0 is a no no. When we try it on our TI-84 calculator, we see ERR:DIVIDE BY 0.
- For any non-zero starting number, starting number / starting number = 1

Using the divide a number by itself process,

every natural number is a black hole.
The process does not work for zero (0).
If \(a\) is a natural number, then \(a / a = 1\).
Black Hole \( Q = 0, \ R = 0 \). Start with any whole number. Divide the whole number by 2 and get a whole number quotient \( Q \) and a whole number remainder \( R \). The remainder \( R \) is 0 or 1.

In this process, we are looking for black hole \( Q = 0, \ R = 0 \). If you divide 0 by 2, you get \( Q = 0, \ R = 0 \). Black hole. Apply the process to the starting number and then apply it to each quotient \( Q \) until you reach the black hole \( Q = 0, \ R = 0 \).

**Start with 0.**

Divide 0 by 2 and get \( Q = 0, \ R = 0 \). Black hole in one application of the process.

**Start with 1.**

Divide 1 by 2 and get \( Q = 0, \ R = 1 \).

Divide 0 by 2 and get \( Q = 0, \ R = 0 \). Black hole in two applications of the process.

**Start with 2.**

Divide 2 by 2 and get \( Q = 1, \ R = 0 \).

Divide 1 by 2 and get \( Q = 0, \ R = 1 \).

Divide 0 by 2 and get \( Q = 0, \ R = 0 \). Black hole in three applications of the process.

**Start with 3.**

Divide 3 by 2 and get \( Q = 1, \ R = 1 \).

Divide 1 by 2 and get \( Q = 0, \ R = 1 \).

Divide 0 by 2 and get \( Q = 0, \ R = 0 \). Black hole in three applications of the process.

**Start with 4.**

Divide 4 by 2 and get \( Q = 2, \ R = 0 \).

Divide 2 by 2 and get \( Q = 1, \ R = 0 \).

Divide 1 by 2 and get \( Q = 0, \ R = 1 \).

Divide 0 by 2 and get \( Q = 0, \ R = 0 \). Black hole in four applications of the process.

Hey! Anyone remember long division?

\[
\begin{array}{cccc}
2) & 0 & \leftarrow \text{quotient} & 2) & 0 & \leftarrow \text{quotient} & 2) & 1 & \leftarrow \text{quotient} & 2) & 1 & \leftarrow \text{quotient} \\
0 & \leftarrow \text{remainder} & 0 & \leftarrow \text{remainder} & 2 & \leftarrow \text{remainder} & 2 & \leftarrow \text{remainder} & 1 & \leftarrow \text{remainder}
\end{array}
\]
Here are tables showing the divide by 2 process for starting numbers 0, 1, 2, 3, and 4.

<table>
<thead>
<tr>
<th>Number</th>
<th>Divide by 2</th>
<th>Q</th>
<th>R</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 / 2</td>
<td>0</td>
<td>0</td>
<td>Black hole $Q = 0, R = 0$</td>
</tr>
<tr>
<td>1</td>
<td>1 / 2</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0 / 2</td>
<td>0</td>
<td>0</td>
<td>Black hole $Q = 0, R = 0$</td>
</tr>
<tr>
<td>2</td>
<td>2 / 2</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1 / 2</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0 / 2</td>
<td>0</td>
<td>0</td>
<td>Black hole $Q = 0, R = 0$</td>
</tr>
<tr>
<td>3</td>
<td>3 / 2</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1 / 2</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0 / 2</td>
<td>0</td>
<td>0</td>
<td>Black hole $Q = 0, R = 0$</td>
</tr>
<tr>
<td>4</td>
<td>4 / 2</td>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2 / 2</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1 / 2</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0 / 2</td>
<td>0</td>
<td>0</td>
<td>Black hole $Q = 0, R = 0$</td>
</tr>
</tbody>
</table>
**Your Turn** Complete this table. The completed table shows that applying the Divide by 2 process to starting number 5 zaps it into black hole $Q = 0, R = 0$ in four applications of the process.

```markdown
<table>
<thead>
<tr>
<th>Number</th>
<th>Divide by 2</th>
<th>Q</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5 / 2</td>
<td>___</td>
<td>___</td>
</tr>
<tr>
<td></td>
<td></td>
<td>___</td>
<td>___</td>
</tr>
<tr>
<td></td>
<td></td>
<td>___</td>
<td>___</td>
</tr>
<tr>
<td></td>
<td></td>
<td>___</td>
<td>___</td>
</tr>
</tbody>
</table>
```

Black hole $Q = 0, R = 0$

**Answers**

```markdown
<table>
<thead>
<tr>
<th>Number</th>
<th>Divide by 2</th>
<th>Q</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5 / 2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2 / 2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1 / 2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0 / 2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
```

Black hole $Q = 0, R = 0$

\[
\begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
5 \\
2
\end{array}
\end{array}
\begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
2 \\
2
\end{array}
\end{array}
\begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
1 \\
2
\end{array}
\end{array}
\begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
1 \\
2
\end{array}
\end{array}
\begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array}
\begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array}
\end{array} \quad \begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
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2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
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2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array}
\end{array} \quad \begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
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2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
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2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
1 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
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2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
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2
\end{array}
\end{array}
\end{array} \quad \begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
1 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
1 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
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\end{array}
\end{array} \quad \begin{array}{c}
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2) \begin{array}{c}
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\end{array}
\end{array} \quad \begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
1 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
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\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
1 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
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\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
1 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
1 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
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\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
1 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
1 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
1 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
2 \downarrow \\
2) \begin{array}{c}
1 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \quad \begin{array}{c}
0 \downarrow \\
2) \begin{array}{c}
0 \\
2
\end{array}
\end{array} \end{array} \end{equation}
Your Turn Complete this table. The completed table shows that applying the Divide by 2 process to starting number 6 plunges it into black hole $Q = 0, R = 0$ in four applications of the process.

<table>
<thead>
<tr>
<th>Number</th>
<th>Divide by 2</th>
<th>$Q$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6 / 2</td>
<td>___</td>
<td>___</td>
</tr>
<tr>
<td></td>
<td></td>
<td>___</td>
<td>___</td>
</tr>
<tr>
<td></td>
<td></td>
<td>___</td>
<td>___</td>
</tr>
<tr>
<td></td>
<td></td>
<td>___</td>
<td>___</td>
</tr>
<tr>
<td></td>
<td></td>
<td>___</td>
<td>___</td>
</tr>
</tbody>
</table>

Black hole $Q = 0, R = 0$

Answers

<table>
<thead>
<tr>
<th>Number</th>
<th>Divide by 2</th>
<th>$Q$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6 / 2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3 / 2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1 / 2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0 / 2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Black hole $Q = 0, R = 0$

Apply the divide by 2 process repeatedly (again and again)
and any whole number
is sucked into black hole $Q = 0, R = 0$. 

\[
\begin{align*}
6 & \div 2 = 3, \quad \text{remainder 0} \\
3 & \div 2 = 1, \quad \text{remainder 1} \\
1 & \div 2 = 0, \quad \text{remainder 1} \\
0 & \div 2 = 0, \quad \text{remainder 0}
\end{align*}
\]
Especially for Teachers and Tutors.

Several black holes of the simple kind relate to important properties of numbers. We summarize.

- **Add Zero Black Hole.** If \( a \) is a whole number, then \( a + 0 = a \) and \( 0 + a = a \).
- **Subtract Zero Black Hole.** If \( a \) is a whole number, then \( a - 0 = a \).
- **Multiply by One Black Hole.** If \( a \) is a whole number, then \( 1 \times a = a \) and \( a \times 1 = a \).
- **Divide by One Black Hole.** If \( a \) is a whole number, then \( a / 1 = a \).
- **Subtract a Number from Itself Black Hole.** If \( a \) is a whole number, then \( a - a = 0 \).
- **Divide a Number by Itself Black Hole.** If \( a \) is a natural number, then \( a / a = 1 \).

Now let’s move on to much more interesting mathemagical black holes. We use them to amaze and amuse students that we tutor. Try them on your students and friends.
The first mathemagical black hole that we learned about was Mathemagical Black Hole 123. We recommend it to amaze and amuse friends and students from first grade up. It requires only counting and catenating (putting together). Catenate 1, 2, and 3 and you get 123. Catenate 12, 34, and 56 and you get 123456. Catenate g, a, m, and e and you get game.

Catenate. Put together, put side by side.
Catenate f, u, and n to get fun.
Catenate 12, 34, and 56 to get 123456.
(catenate is also called concatenate)

We will demonstrate the count & catenate process by applying it to the number 123.

Start with 123.
Count the number of even digits. The number 123 has 1 even digit.
Count the number of odd digits. The number 123 has 2 odd digits.
Count the total number of digits. The number 123 has 3 digits.
Catenate (put together) the three numbers in the order: even odd total. You get 123. Voila! Black hole 123.

Below is a handy table that shows the count & catenate process applied to the number 123.

<table>
<thead>
<tr>
<th>Start with 123</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>123</td>
</tr>
<tr>
<td>123</td>
</tr>
</tbody>
</table>

The number 123 is a mathemagical black hole using the count & catenate process. Now here is a boggler: Start with any natural number, apply this process repeatedly, and you will end up in black hole 123.

Even digits are 0, 2, 4, 6, and 8.
Odd digits are 1, 3, 5, 7, and 9.
Hey! To play this black hole game, all you need to know is how to count and catenate (put together) three counting numbers. That’s why we call it the count & catenate process.

Can first-grade students count and catenate? If yes, then they can play Black Hole 123.

Let’s apply the count & catenate process to the numbers 213 and 321.

**Start with 213.**

Count the number of even digits. The number 213 has 1 even digit.
Count the number of odd digits. The number 213 has 2 odd digits.
Count the total number of digits. The number 213 has 3 digits.
Catenate the three numbers in the order: **even odd total**. You get 123. The number 213 collapses into black hole 123 in one application of the count & catenate process.

<table>
<thead>
<tr>
<th>Start with 213</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>213</td>
</tr>
<tr>
<td>123</td>
</tr>
</tbody>
</table>

**Start with 321.**

Count the number of even digits. The number 321 has 1 even digit.
Count the number of odd digits. The number 321 has 2 odd digits.
Count the total number of digits. The number 321 has 3 digits.
Catenate the three numbers in the order: **even odd total**. You get 123. The number 321 moseys into black hole 123 in one application of the count & catenate process.

<table>
<thead>
<tr>
<th>Start with 321</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>321</td>
</tr>
<tr>
<td>123</td>
</tr>
</tbody>
</table>

The numbers 123, 213, and 321 have the same digits arranged in different order. They are **permutations**. The numbers 132, 231, and 312 are also permutations with the digits 1, 2, and 3.
Your Turn  Apply the count & catenate process to the number 231 *before* you peek at our answers down yonder. Complete the numbered steps and the table.

1. Count the number of even digits. The number 231 has ___ even digit(s).
2. Count the number of odd digits. The number 231 has ___ odd digit(s).
3. Count the total number of digits. The number 231 has ___ digits.

4. Catenate your three answers above. ______

Did 231 plop into black hole 123? ______

<table>
<thead>
<tr>
<th>Start with 231</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>231</td>
</tr>
<tr>
<td>______</td>
</tr>
</tbody>
</table>

Answers

1. Count the number of even digits. The number 231 has 1 even digit.
2. Count the number of odd digits. The number 231 has 2 odd digits.
3. Count the total number of digits. The number 231 has 3 digits.
4. Catenate your three answers above. 123

Did 231 plop into black hole 123? Yes

<table>
<thead>
<tr>
<th>Start with 231</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>231</td>
</tr>
<tr>
<td>123</td>
</tr>
</tbody>
</table>

On the next page we will apply the count and catenate process to the numbers 357 and 246.

- 357 has no even digits. The number of even digits is 0.
- 246 has no odd digits. The number of odd digits is 0.

Think about how it might go, and then amble on down to the next page.
Start with 357.
Count the number of even digits. The number 357 has 0 even digits.
Count the number of odd digits. The number 357 has 3 odd digits.
Count the total number of digits. The number 357 has 3 digits.
Catenate 0, 3, and 3 and get 033. **Be sure to include the leading 0 when you count & catenate.**
Count the number of even digits. The number 033 has 1 even digit.
Count the number of odd digits. The number 033 has 2 odd digits.
Count the total number of digits. The number 033 has 3 digits.
Catenate 1, 2, and 3 and get 123. The number 357 is slurped into black hole 123 in two applications of the count & catenate process.

<table>
<thead>
<tr>
<th>Start with 357</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>357</td>
</tr>
<tr>
<td>033</td>
</tr>
<tr>
<td>123</td>
</tr>
</tbody>
</table>

Q. Permutations: 357, 375, 537, 573, 735, and 753. If you apply the count & catenate process to one of these permutations, will it be gobbled up by black hole 123?

Start with 246.
Count the number of even digits. The number 246 has 3 even digits.
Count the number of odd digits. The number 246 has 0 odd digits.
Count the total number of digits. The number 246 has 3 digits.
Catenate 3, 0, and 3 and get 303. Apply the process to 303.
Count the number of even digits. The number 303 has 1 even digit.
Count the number of odd digits. The number 303 has 2 odd digits.
Count the total number of digits. The number 303 has 3 digits.
Catenate 1, 2, and 3 and get 123. The number 246 is zoomed into black hole 123 in two applications of the count & catenate process.

<table>
<thead>
<tr>
<th>Start with 246</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>246</td>
</tr>
<tr>
<td>303</td>
</tr>
<tr>
<td>123</td>
</tr>
</tbody>
</table>
Your Turn Apply the count & catenate process to the number 1234. Complete the table.

<table>
<thead>
<tr>
<th>number</th>
<th># even</th>
<th># odd</th>
<th>total #</th>
</tr>
</thead>
<tbody>
<tr>
<td>1234</td>
<td>___</td>
<td>___</td>
<td>___</td>
</tr>
<tr>
<td>224</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>303</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>123</td>
<td></td>
<td></td>
<td>Black hole 123? Yes</td>
</tr>
</tbody>
</table>

Answers

<table>
<thead>
<tr>
<th>number</th>
<th># even</th>
<th># odd</th>
<th>total #</th>
</tr>
</thead>
<tbody>
<tr>
<td>1234</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>224</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>303</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>123</td>
<td></td>
<td></td>
<td>Black hole 123? Yes</td>
</tr>
</tbody>
</table>

Together we have applied the count & catenate process to 3-digit numbers and one 4-digit number. Yup, they all got pulled into black hole 123. Does the process work for 1-digit numbers and 2-digit numbers? Let's find out.

<table>
<thead>
<tr>
<th>number</th>
<th># even</th>
<th># odd</th>
<th>total #</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>101</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>123</td>
<td></td>
<td></td>
<td>Black hole 123</td>
</tr>
</tbody>
</table>
Your Turn Apply the count & catenate process to the 2-digit number 11.

Start with 11

<table>
<thead>
<tr>
<th>number</th>
<th># even</th>
<th># odd</th>
<th>total #</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>123</td>
<td></td>
<td></td>
<td>Black hole 123?</td>
</tr>
</tbody>
</table>

Answers

Start with 11

<table>
<thead>
<tr>
<th>number</th>
<th># even</th>
<th># odd</th>
<th>total #</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>022</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>303</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>123</td>
<td></td>
<td></td>
<td>Black hole 123? Yes</td>
</tr>
</tbody>
</table>
Let's boldly try a much larger number.

**Start with 1223334444.**

Count the number of even digits. The number 1223334444 has 6 even digits.
Count the number of odd digits. The number 1223334444 has 4 odd digits.
Count the total number of digits. The number 1223334444 has 10 digits.
Catenate 6, 4, and 10 to get 6410. Apply the count & catenate process to 6410.

Count the number of even digits. The number 6410 has 3 even digits.
Count the number of odd digits. The number 6410 has 1 odd digit.
Count the total number of digits. The number 6410 has 4 digits.
Catenate 3, 1, and 4 to get 314. Apply the count & catenate process to 314.

Count the number of even digits. The number 314 has 1 even digit.
Count the number of odd digits. The number 314 has 2 odd digits.
Count the total number of digits. The number 314 has 3 digits.
Catenate 1, 2, and 3 to get 123. Black hole.

The number 1223334444 was eased into black hole 123 in three applications of the count & catenate process. Here are the steps in handy table form.

<table>
<thead>
<tr>
<th>number</th>
<th># even</th>
<th># odd</th>
<th>total #</th>
</tr>
</thead>
<tbody>
<tr>
<td>1223334444</td>
<td>6</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>6410</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>314</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>123</td>
<td></td>
<td></td>
<td>Black hole.</td>
</tr>
</tbody>
</table>

Serendipity! The table shows the action in a compact way compared to the verbose description above the table. It is easy to see every step in the count & catenate process. We like it. What do you think?
Your Turn Apply the count & catenate process to the number 123456789.

<table>
<thead>
<tr>
<th>number</th>
<th># even</th>
<th># odd</th>
<th>total #</th>
</tr>
</thead>
<tbody>
<tr>
<td>123456789</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>123</td>
<td></td>
<td></td>
<td>Black hole.</td>
</tr>
</tbody>
</table>

Start with 123456789

Answers

<table>
<thead>
<tr>
<th>number</th>
<th># even</th>
<th># odd</th>
<th>total #</th>
</tr>
</thead>
<tbody>
<tr>
<td>123456789</td>
<td>4</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>459</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>123</td>
<td></td>
<td></td>
<td>Black hole.</td>
</tr>
</tbody>
</table>

Still Your Turn – Investigate

You can find Black Hole 123 play sheets down yonder on page 26.

1. Start with 1234567890. Arrive at black hole 123 in three applications of the count & catenate process. Show the steps in a table.

2. Find a number that requires four applications of the count & catenate process to zap it into black hole 123.

3. Find a number that requires five applications of the count & catenate process to whirl it into black hole 123.

4. Find a number that requires $n$ applications of the count & catenate process to whirl it into black hole 123. You choose the value of $n$. 
Questions

1. There are six ways to arrange the digits 1, 2, and 3 into a 3-digit number. These ways are called permutations.

   The permutations are 123, 132, 213, 231, 312, and 321.

   You know that 123 is a black hole. How many applications of the count & catenate process will it take to stuff 132, 213, 231, 312, or 321 into black hole 123?

2. The permutations of 235 are 235, 253, 325, 352, 523, and 532. The number 235 homes in on black hole 123 in one application of the count & catenate process. How many applications of the count & catenate process will it take to plunge 253, 325, 352, or 532 into black hole 123?

3. The permutations of 357 are 357, 375, 537, 573, 735, and 753. The number 357 arrives at black hole 123 in two applications of the count & catenate process. How many applications of the count & catenate process will it take to drag 375, 537, 573, or 753 into black hole 123?

4. Examine these numbers: 101, 121, 141, 161, 181, 191. For each of these numbers, how many applications of the count & catenate process will it take to slurp it into black hole 123?

5. Examine these numbers: 212, 414, 616, 818. For each of these numbers, how many applications of the count & catenate process will it take to suck it into black hole 123?

Answers

1. One application. Each of these numbers has 1 even digit, 2 odd digits, and 3 total digits. Catenate 1, 2, and 3 to get 123. Black hole.

2. One application. Each of these numbers has 1 even digit, 2 odd digits, and 3 total digits. Catenate 1, 2, and 3 to get 123. Black hole.

3. Two applications. Each permutation of 357 starts out with 0 even digits, 3 odd digits, and 3 total digits. Catenate 0, 3, and 3 to get 033. The number 033 has 1 even digit, 2 odd digits, and 3 total digits. Catenate 1, 2, and 3 to get 123. Black hole.

4. One application. Each of these numbers has 1 even digit, 2 odd digits, and 3 total digits. Catenate 1, 2, and 3 to get 123. Black hole.

5. Two applications. Each of these numbers has 2 even digits, 1 odd digit, and 3 total digits. Catenate 2, 1, and 3 to get 213. The number 213 has 1 even digit, 2 odd digits, and 3 total digits. Catenate 1, 2, and 3 to get 123. Black hole.
Investigations An investigation can require a lot of thinking and a bit of work. If you are a classroom teacher, you can form teams of students, and assign each team part of the work.

Any 3-digit number can be transformed into black hole 123 by one or more applications of the count & catenate process.

Three-digit numbers are the numbers 100, 101, 102, and so on up to 998 and 999. Form nine teams and have each team investigate 100 numbers. Team 1 investigates 100 through 199, Team 2 takes on 200 through 299, Team 3 studies 300 through 399, and so on.

1. Make a list of the 3-digit numbers that are transformed into black hole 123 by one application of the count & catenate process. How many 3-digit numbers are transformed into black hole 123 by one application of the count & catenate process?

2. Make a list of the 3-digit numbers that are transformed into black hole 123 by two applications of the count & catenate process. How many 3-digit numbers are transformed into black hole 123 by two applications of the count & catenate process?

3. Are there any 3-digit numbers that require three or more applications of the count & catenate process in order to become black hole 123?

4. Repeat the above investigations for 4-digit numbers, 5-digit numbers, or more-digit numbers.

We like to do black hole alakazams with students and tutees. They love it because they can use black holes to amaze and amuse their friends. We suggest that they apply the count & catenate process to one or more of their favorite numbers. Here are some of our favorite numbers:

Speed of light in meters per second: 299,792,458
One light year in kilometers: 9,460,523,256,000
Average distance of the Earth from the Sun in kilometers: 149,600,000
Circumference of the Earth at the equator in kilometers: 40,074
Length of an Earth day in seconds: 86,400
Length of an Earth week in seconds: 604,800
Length of an Earth year in seconds using 365 days per year: 31,536,000
A more exact value for the length of an Earth year is 365.242 days. Using this value, the length of an Earth year in seconds is 31,556,909.
Lottery. Number of ways to select 6 different numbers from the numbers 1 to 52, inclusive: 20,358,520.
One googol is equal to 10 to the power 100 (10^{100}). Write it as 1 followed by 100 zeros.
The palindromic numbers 11, 121, 12321, 1234321, 12345654321, 1234567654321, 123456787654321, and 12345678987654321. These numbers are the squares of the numbers 11, 111, 1111, and so on.
The number 1223334444555556666667777777888888889999999999.
Black Hole 123 Play Sheets | **TOC**

Start with __________________________

<table>
<thead>
<tr>
<th>number</th>
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<th># odd</th>
<th>total #</th>
</tr>
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</table>

Start with __________________________

<table>
<thead>
<tr>
<th>number</th>
<th># even</th>
<th># odd</th>
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<td></td>
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<td></td>
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</tr>
</tbody>
</table>

26
Mathemagical Black Holes 99 & 1089 | TOC

The mathemagical numbers 99 and 1089 have many interesting alakazams. Before we delve into their black hole properties, we will meander about other properties of 99 and 1089.

The reverse of a number is the number you get by reading the digits of the given number in reverse, backwards, from right to left instead of left to right. If a number and its reverse are the same number, then the number is a palindrome.

- The reverse of 123 is 321.
- The reverse of 99 is 99. [99 is a palindrome.]
- The reverse of 1089 is 9801.
- The reverse of 9801 is 1089.

The reverse of 99 is 99, so 99 is a palindrome. Read 99 backwards and what do you get? 99. Multiply 99 by 9 and you get 1089. \(9 \times 99 = 1089\).

The square of 99 is 9801, which is the reverse of 1089. \([99^2 = 9801]\)

9801 is the reverse of 1089 and 1089 is the reverse of 9801.

The sum of the digits of 99 is 18, and the sum of the digits of 18 is 9.

The sum of the digits of 1089 is 18, and the sum of the digits of 18 is 9.

The number 99 is a black hole for 2-digit numbers that are not palindromes using the process described below.

1. Start with any 2-digit number that is not a palindrome. [Cannot be 11, 22, 33, …, 99.]
2. Reverse the starting number. Write it backwards.
3. You now have two numbers. Subtract the lesser number from the greater number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 99.

The number 1089 is a black hole for 3-digit numbers that are not palindromes using the process described below.

1. Start with any 3-digit number that is not a palindrome. [Cannot be 101, 202, 383, 999, ….]
2. Reverse the starting number. Write it backwards.
3. You now have two numbers. Subtract the lesser number from the greater number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 1089.

Onward to examples and Your Turn activities, first for 99 and then for 1089.
Black Hole 99

The Black Hole 99 Process
1. Start with any 2-digit number that is not a palindrome. Start with 73.
2. Reverse the starting number. Write it backwards. The reverse of 73 is 37.
3. Subtract the lesser number from the greater number. 73 – 37 = 36
4. Reverse the result of the subtraction in Step 3. The reverse of 36 is 63.
5. Add the results of Step 3 and Step 4. Black hole 99. 36 + 63 = 99

Example
Start with 73.
The reverse of 73 is 37.
73 – 37 = 36
The reverse of 36 is 63.
36 + 63 = 99

Your Turn Apply the Black Hole 99 process to 82.
1. Start with 82.
2. Reverse the starting number. Write it backwards. The reverse of 82 is ___.
3. Subtract the lesser number from the greater number. ____ – ____ = ____
4. Reverse the result of the subtraction in Step 3. The reverse of ____ is ____.
5. Add the results of Step 3 and Step 4. ____ + ____ = ____
Did you get 99? If not, check your calculations.

Answers
1. Start with 82. Start with 82.
2. Reverse the starting number. Write it backwards. The reverse of 82 is 28.
3. Subtract the lesser number from the greater number. 82 – 28 = 54
4. Reverse the result of the subtraction in Step 3. The reverse of 54 is 45.
5. Add the results of Step 3 and Step 4. Black hole 99. 54 + 45 = 99
In our example and in the **Your Turn** exercise, the result in Step 3 is a 2-digit number. For some choices of starting number, the result in Step 3 may be the 1-digit number 9. In this case, add a leading zero to make it 09. Follow along as we do an example.

### The Black Hole 99 Process

1. Start with 23.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number. The result is the 1-digit number 9. Add a leading zero in order to make a 2-digit number.
   
   \[
   \begin{align*}
   32 - 23 &= 9 \\
   32 - 23 &= 09
   \end{align*}
   \]
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 99.

### Example

- Start with 23.
- The reverse of 23 is 32.
- \(32 - 23 = 9\)
- \(32 - 23 = 09\)
- The reverse of 09 is 90.
- \(09 + 90 = 99\)

### Your Turn

Apply the Black Hole 99 process to 67.

1. Start with 67.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number. If the result is the 1-digit number 9, add a leading zero in order to make a 2-digit number.
   
   \[
   \begin{align*}
   \text{____} - \text{____} &= \text{____} \\
   \text{____} - \text{____} &= \text{____}
   \end{align*}
   \]
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 99.

Did you get 99? If not, check your calculations.

### Answers

1. Start with 67.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number. The result is the 1-digit number 9. Add a leading zero in order to make a 2-digit number.
   
   \[
   \begin{align*}
   76 - 67 &= 9 \\
   76 - 67 &= 09
   \end{align*}
   \]
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 99.

\(09 + 90 = 99\)
**Conjecture.** If the digits of a 2-digit number differ by 1, then the digits of the reverse of the number also differ by 1. The result of subtracting the lesser number from the greater number is 9.

<table>
<thead>
<tr>
<th>Number</th>
<th>Reverse</th>
<th>Greater</th>
<th>Lesser</th>
<th>Greater − lesser</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>01</td>
<td>10</td>
<td>01</td>
<td>10 − 01 = 9</td>
</tr>
<tr>
<td>12</td>
<td>21</td>
<td>21</td>
<td>12</td>
<td>21 − 12 = 9</td>
</tr>
<tr>
<td>43</td>
<td>34</td>
<td>43</td>
<td>34</td>
<td>43 − 34 = 9</td>
</tr>
<tr>
<td>89</td>
<td>98</td>
<td>98</td>
<td>89</td>
<td>98 − 89 = 9</td>
</tr>
</tbody>
</table>

**Conjecture.** If the digits of a 2-digit number differ by 2 or more, then the digits of the reverse of the number also differ by 2 or more. The result of subtracting the lesser number from the greater number is a 2-digit number and the sum of its digits is 9.

<table>
<thead>
<tr>
<th>Number</th>
<th>Reverse</th>
<th>Greater</th>
<th>Lesser</th>
<th>Greater − lesser</th>
<th>Sum of digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>02</td>
<td>20</td>
<td>02</td>
<td>20 − 02 = 18</td>
<td>1 + 8 = 9</td>
</tr>
<tr>
<td>14</td>
<td>41</td>
<td>41</td>
<td>14</td>
<td>41 − 14 = 27</td>
<td>2 + 7 = 9</td>
</tr>
<tr>
<td>62</td>
<td>26</td>
<td>62</td>
<td>26</td>
<td>62 − 26 = 36</td>
<td>3 + 6 = 9</td>
</tr>
<tr>
<td>38</td>
<td>83</td>
<td>83</td>
<td>38</td>
<td>83 − 38 = 45</td>
<td>4 + 5 = 9</td>
</tr>
<tr>
<td>90</td>
<td>09</td>
<td>90</td>
<td>09</td>
<td>90 − 09 = 81</td>
<td>8 + 1 = 9</td>
</tr>
</tbody>
</table>

The Black Hole 99 process works for 2-digit numbers that are not palindromes. What happens if we apply it to the palindrome 77?

**The Black Hole 99 Process**

1. Start with 77. [A palindrome]
2. Reverse the starting number. Write it backwards. The reverse of 77 is 77.
3. Subtract the lesser number from the greater number. The result is the 1-digit number 0. Add a leading zero in order to make a 2-digit number.
4. Add the results of Step 3 and Step 4. Black hole 0?
5. The process slurps a 2-digit palindrome into black hole 0. This is equivalent to the Subtract a Number from Itself Black Hole that we explored in Black Holes of the Simple Kind.

**Palindrome Example**

| Start with 77. | Reverse of 77 is 77. | Subtract the lesser number from the greater number. The result is the 1-digit number 0. Add a leading zero in order to make a 2-digit number. | The reverse of 00 is 00. | 00 + 00 = 00 = 0 |
|----------------|----------------------|---------------------------------------------------------------------------------------------------------------------------------|------------------------|
| 77 − 77 = 0    | 77 − 77 = 0          | 77 − 77 = 0                                                                                                                     | 00 + 00 = 00 = 0       |
**Black Hole 1089**

**The Black Hole 1089 Process**

1. Start with any 3-digit number that is not a palindrome. Start with 123.
2. Reverse the starting number. Write it backwards. The reverse of 123 is 321.
3. Subtract the lesser number from the greater number. 321 – 123 = 198
4. Reverse the result of the subtraction in Step 3. The reverse of 198 is 891.
5. Add the results of Step 3 and Step 4. Black hole 1089. 198 + 981 = 1089

**Example**

Start with 123.

The reverse of 123 is 321.

321 – 123 = 198

The reverse of 198 is 891.

198 + 981 = 1089

**Your Turn** Apply the Black Hole 1089 process to 235.

2. Reverse the starting number. Write it backwards. The reverse of 235 is 532.
3. Subtract the lesser number from the greater number. 532 – 235 = 297
4. Reverse the result of the subtraction in Step 3. The reverse of 297 is 792.
5. Add the results of Step 3 and Step 4. Black hole 1089. 297 + 792 = 1089

Did you get 1089? If not, check your calculations.

**Answers**

2. Reverse the starting number. Write it backwards. The reverse of 235 is 532
3. Subtract the lesser number from the greater number. 532 – 235 = 297
4. Reverse the result of the subtraction in Step 3. The reverse of 297 is 792.
5. Add the results of Step 3 and Step 4. Black hole 1089. 297 + 792 = 1089

**Did you notice?**

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>198 is divisible by 9. 198 / 9 = 22.</td>
<td>297 is divisible by 9. 297 / 9 = 33.</td>
</tr>
<tr>
<td>The sum of the digits of 198 is 18 and the sum of the digits of 18 is 9.</td>
<td>The sum of the digits of 297 is 18 and the sum of the digits of 18 is 9.</td>
</tr>
</tbody>
</table>
In our example and in the **Your Turn** exercise, the result in Step 3 is a 3-digit number. For some choices of starting number, the result in Step 3 may be the 2-digit number 99. In this case, add a leading zero to make it 099. Follow along as we do an example.

### The Black Hole 1089 Process

1. Start with 556.
2. Reverse the starting number. Write it backwards. 
3. Subtract the lesser number from the greater number. The result is the 2-digit number 99. Add a leading zero in order to make a 3-digit number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 1089.

#### Example

- Start with 556.
- The reverse of 556 is 655.
- \[655 - 556 = 99\]
- \[655 - 556 = 099\]
- The reverse of 099 is 990.
- \[099 + 990 = 1089\]

### Your Turn

Apply the Black Hole 1089 process to 344.

1. Start with 344.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number. If the result is the 2-digit number 99, add a leading digit in order to make a 3-digit number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 1089.

#### Example

- Start with 344.
- The reverse of 344 is 443.
- \[443 - 344 = 99\]
- \[443 - 344 = 099\]
- The reverse of 099 is 990.
- \[099 + 990 = 1089\]

### Answers

1. Start with 344.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number. The result is the 2-digit number 99. Add a leading digit in order to make a 3-digit number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 1089.

#### Example

- Start with 344.
- The reverse of 344 is 443.
- \[443 - 344 = 99\]
- \[443 - 344 = 099\]
- The reverse of 099 is 990.
- \[099 + 990 = 1089\]
Practice your newly acquired Black Hole 1089 skills by romping through these **Your Turns**.

### Example
Start with 642.

1. Start with 642.
2. Reverse the starting number. Write it backwards. 
   - The reverse of 642 is _____.
3. Subtract the lesser number from the greater number. 
   - _____ – _____ = _____
4. Reverse the result of the subtraction in Step 3. 
   - The reverse of _____ is _____.
5. Add the results of Step 3 and Step 4. 
   - _____ + _____ = _____

### Answers
1. Start with 642.
2. Reverse the starting number. Write it backwards. 
   - The reverse of 642 is 246.
3. Subtract the lesser number from the greater number. 
   - 642 – 246 = 396
4. Reverse the result of the subtraction in Step 3. 
   - The reverse of 396 is 693.
5. Add the results of Step 3 and Step 4. Black hole 1089. 
   - 396 + 693 = 1089

### Your Turn
Apply the Black Hole 1089 process to 642.

1. Start with 642.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4.

### Example
Start with 951.

1. Start with 951.
2. Reverse the starting number. Write it backwards. 
   - The reverse of 951 is 159.
3. Subtract the lesser number from the greater number. 
   - 951 – 159 = 792
4. Reverse the result of the subtraction in Step 3. 
   - The reverse of 792 is 297.
5. Add the results of Step 3 and Step 4. Black hole 1089. 
   - 792 + 297 = 1089

### Answers
1. Start with 951.
2. Reverse the starting number. Write it backwards. 
   - The reverse of 951 is 159.
3. Subtract the lesser number from the greater number. 
   - 951 – 159 = 792
4. Reverse the result of the subtraction in Step 3. 
   - The reverse of 792 is 297.
5. Add the results of Step 3 and Step 4. Black hole 1089. 
   - 792 + 297 = 1089
### Your Turn
Apply the Black Hole 1089 process to 667.

1. Start with 667.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number. If the result is a 2-digit number, add a leading zero in order to make a 3-digit number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 1089.

### Example
Start with 667.

1. Start with 667.
2. The reverse of 667 is _____.
3. The reverse of 099 is 990.
4. 099 + 990 = 1089

### Your Turn
Apply the Black Hole 1089 process to 998.

1. Start with 998.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4.

### Example
Start with 998.

1. Start with 998.
2. The reverse of 998 is 990.
3. 998 – 899 = 099
4. 099 + 990 = 1089
**Conjecture.** If the first and last digits of a 3-digit number and its reverse differ by 1, then the result of subtracting the lesser number from the greater number is 99.

<table>
<thead>
<tr>
<th>Number</th>
<th>Reverse</th>
<th>Greater</th>
<th>Lesser</th>
<th>Greater – lesser</th>
</tr>
</thead>
<tbody>
<tr>
<td>102</td>
<td>201</td>
<td>201</td>
<td>102</td>
<td>201 – 102 = 99</td>
</tr>
<tr>
<td>473</td>
<td>374</td>
<td>473</td>
<td>374</td>
<td>473 – 374 = 99</td>
</tr>
<tr>
<td>998</td>
<td>899</td>
<td>998</td>
<td>899</td>
<td>998 – 899 = 99</td>
</tr>
</tbody>
</table>

**Conjecture.** If the first and last digits of a 3-digit number and its reverse differ by 2 or more, then the result of subtracting the lesser number from the greater number is a 3-digit number and the sum of its digits is 18.

<table>
<thead>
<tr>
<th>Number</th>
<th>Reverse</th>
<th>Greater</th>
<th>Lesser</th>
<th>Greater – lesser</th>
<th>Sum of digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>002</td>
<td>200</td>
<td>002</td>
<td>200 – 002 = 198</td>
<td>1 + 9 + 8 = 18</td>
</tr>
<tr>
<td>134</td>
<td>431</td>
<td>431</td>
<td>134</td>
<td>431 – 134 = 297</td>
<td>2 + 9 + 7 = 18</td>
</tr>
<tr>
<td>612</td>
<td>216</td>
<td>612</td>
<td>216</td>
<td>612 – 216 = 396</td>
<td>3 + 9 + 6 = 18</td>
</tr>
<tr>
<td>368</td>
<td>863</td>
<td>863</td>
<td>368</td>
<td>863 – 368 = 495</td>
<td>4 + 9 + 5 = 18</td>
</tr>
<tr>
<td>900</td>
<td>009</td>
<td>900</td>
<td>009</td>
<td>900 – 009 = 891</td>
<td>8 + 9 + 1 = 18</td>
</tr>
</tbody>
</table>

The Black Hole 1089 process works for 3-digit numbers that are not palindromes. What happens if we apply it to the palindrome 777?

### The Black Hole 1089 Process
1. Start with 777. [A palindrome]
2. Reverse the starting number. Write it backwards. The reverse of 777 is 777.
3. Subtract the lesser number from the greater number. The result is the 1-digit number 0. Add two leading zeros in order to make a 3-digit number. 777 – 777 = 0
   
   777 – 777 = 000
4. Reverse the result of the subtraction in Step 3. The reverse of 000 is 000.
5. Add the results of Step 3 and Step 4. Black hole 0? 000 + 000 = 000 = 0

The process slurps a 3-digit palindrome into black hole 0. This is equivalent to the **Subtract a Number from Itself** black hole that we explored in the chapter **Black Holes of the Simple Kind**.

**Be prepared!** What if your tutee asks, "Why does it work?" We recommend a visit to this handy Web site:
- Why is it always 1089? [http://www.mathsisfun.com/1089algebra.html](http://www.mathsisfun.com/1089algebra.html)
Mathemagical Alakazams

The Black Hole 99 Process
1. Start with _____.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number. If the result is the 1-digit number 9, add a leading zero in order to make a 2-digit number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 99.

Example
1. Start with _____.
The reverse of _____ is _____.
   _____ - _____ = _____
   _____ - _____ = _____
The reverse of _____ is _____.
   _____ + _____ = _____

The Black Hole 1089 Process
1. Start with _____.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number. If the result is the 2-digit number 99, add a leading zero in order to make a 3-digit number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 1089.

Example
1. Start with _____.
The reverse of _____ is _____.
   _____ - _____ = __-
   _____ - _____ = _____
The reverse of _____ is _____.
   _____ + _____ = _____

The Black Hole 1089 Process
1. Start with _____.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 1089.

Example
1. Start with _____.
The reverse of _____ is _____.
   _____ - _____ = ______
The reverse of _____ is _____.
   ___-__ + ______ = ______
What About 4-Digit Numbers?

We have described a black hole process for 2-digit numbers and 3-digit numbers.
- The process zooms any 2-digit number that is not a palindrome into black hole 99.
- The process zaps any 3-digit number that is not a palindrome into black hole 1089.

What might happen if we apply the process to 4-digit numbers? Easy to find out – just do it.
Hey! Apply the process to the 4-digit number 1089.

The Black Hole Process

1. Start with 1089.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 10890?

Example

Start with 1089.
The reverse of 1089 is 9801.
9801 \( - 1089 = 8712 \)
The reverse of 8712 is 2178.
8712 + 2178 = 10890

Is 10890 a black hole for 4-digit numbers? Apply the process to the 4-digit number 1234.

The Black Hole Process

Start with 1234.
1. Reverse the starting number. Write it backwards.
2. Subtract the lesser number from the greater number.
3. Reverse the result of the subtraction in Step 3.
4. Add the results of Step 3 and Step 4. Black hole 10890?

Example

Start with 1234.
The reverse of 1234 is 4321.
4321 \( - 1234 = 3087 \)
The reverse of 3087 is 7803.
3087 + 7803 = 10890

Is 10890 a black hole for 4-digit numbers? It works for the 4-digit numbers 1089 and 1234.

Question 1. How many 4-digit numbers are there? 4-digit numbers are 1000, 1001, 1002, ..., 9997, 9998, and 9999.

Question 2. How many 4-digit numbers are palindromes? Palindromic 4-digit numbers are 1001, 1111, 1221, ..., 9991, 2002, 2112, et cetera, et cetera, up to 9889 and 9999.

Question 3. How many 4-digit numbers are not palindromes?
Our sample size is small, only two 4-digit numbers. We need to gather evidence by applying the process to more 4-digit numbers. We tried a bunch of numbers and found some very interesting outcomes.

**The Black Hole Process**

2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number. The result is the 3-digit number 999. Add a leading zero in order to make it a 4-digit number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 10989?

**Example**


The reverse of 2001 is 1002.

2001 – 1002 = 999

2001 – 1002 = 0999

The reverse of 0999 is 9990.

0999 + 9990 = 10989

The process zapped 2001 into 10989, not into 10890. Have we found two black hole destinations for 4-digit numbers? We continue investigating.

**The Black Hole Process**

1. Start with 5618.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 9999?

**Example**

Start with 5618.

The reverse of 5618 is 8165.

8165 – 5618 = 2547

The reverse of 2547 is 7452.

2547 + 7452 = 9999

Black holes are proliferating! We have found three black holes: 10890, 10989, and 9999. Next we will apply the process to 8128.

**The Black Hole Process**

1. Start with 8128.
2. Reverse the starting number. Write it backwards.
3. Subtract the lesser number from the greater number. The result is the 2-digit number 90. Add two leading zeros in order to make it a 4-digit number.
4. Reverse the result of the subtraction in Step 3.
5. Add the results of Step 3 and Step 4. Black hole 990?

**Example**

Start with 8128.

The reverse of 8128 is 8218.

8218 – 8128 = 90

8218 – 8128 = 0090

The reverse of 0090 is 0900.

0090 + 0900 = 990 = 990

Another black hole? Will other 4-digit numbers fall into this black hole? Are there more black holes? It is **your turn** to investigate.
Your Turn: Apply the Black Hole process to 9876.

1. Start with 9876.
2. Reverse the starting number. Write it backwards. The reverse of 9876 is _____.
3. Subtract the lesser number from the greater number. _____ – _____ = _____
4. Reverse the result of the subtraction in Step 3. The reverse of _____ is _____.
5. Add the results of Step 3 and Step 4. _____ + _____ = _____

Example

1. Start with 9876.
2. Reverse the starting number. Write it backwards. The reverse of 9876 is 6789.
3. Subtract the lesser number from the greater number. 9876 – 6789 = 3087
4. Reverse the result of the subtraction in Step 3. The reverse of 3087 is 7803.
5. Add the results of Steps 3 and 4. Black hole 10890. 3087 + 7802 = 10890

Answers

1. Start with 9876.
2. Reverse the starting number. Write it backwards. The reverse of 9876 is 6789.
3. Subtract the lesser number from the greater number. 9876 – 6789 = 3087
4. Reverse the result of the subtraction in Step 3. The reverse of 3087 is 7803.
5. Add the results of Steps 3 and 4. Black hole 10890. 3087 + 7802 = 10890

The digits of 9876 decrease by 1 from left to right. The reverse of 9876 is 6789. The digits of 6789 increase by 1 from left to right. We applied the process to other numbers of this type.

<table>
<thead>
<tr>
<th>Number</th>
<th>Reverse</th>
<th>Fell into black hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>8765</td>
<td>5678</td>
<td>10890</td>
</tr>
<tr>
<td>7654</td>
<td>4567</td>
<td>10890</td>
</tr>
<tr>
<td>6543</td>
<td>3456</td>
<td>10890</td>
</tr>
<tr>
<td>5432</td>
<td>2345</td>
<td>10890</td>
</tr>
<tr>
<td>4321</td>
<td>1234</td>
<td>10890</td>
</tr>
</tbody>
</table>

We applied the process to 7531, 9753, 6420, and 8642. Here are the results for 7531 and 6420.

<table>
<thead>
<tr>
<th>Number</th>
<th>Reverse</th>
<th>Fell into black hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>7531</td>
<td>1357</td>
<td>10890</td>
</tr>
<tr>
<td>6420</td>
<td>0246</td>
<td>10890</td>
</tr>
</tbody>
</table>

Apply the process to 9753 and 8642. Do they fall into black hole 10890?
Your Turn Apply the Black Hole process to 9337.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Start with 9337.</td>
</tr>
<tr>
<td>2.</td>
<td>Reverse the starting number. Write it backwards. The reverse of 9337 is ______.</td>
</tr>
<tr>
<td>3.</td>
<td>Subtract the lesser number from the greater number. _____ – _____ = _____</td>
</tr>
<tr>
<td>4.</td>
<td>Reverse the result of the subtraction in Step 3. The reverse of _____ is ______.</td>
</tr>
<tr>
<td>5.</td>
<td>Add the results of Step 3 and Step 4. _____ + _____ = _____</td>
</tr>
</tbody>
</table>

**Example**

1. Start with 9337.
2. The reverse of 9337 is 7339.
3. 9337 – 7339 = 1998
4. The reverse of 1998 is 8991.
5. 1998 + 8991 = 10989

**Answers**

1. Start with 9337.
2. The reverse of 9337 is 7339.
3. 9337 – 7339 = 1998
4. The reverse of 1998 is 8991.
5. 1998 + 8991 = 10989

We applied the process to 1004, 2003, 3221, 4887, 5111, 6442, 7332, 8772, 9222, and 9331. They all fell into black hole 10989. Here is the process applied to 7332.

**The Black Hole Process**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Start with 7332.</td>
</tr>
<tr>
<td>2.</td>
<td>Reverse the starting number. Write it backwards. The reverse of 7332 is 2337.</td>
</tr>
<tr>
<td>3.</td>
<td>Subtract the lesser number from the greater number. 7332 – 2337 = 4995</td>
</tr>
<tr>
<td>4.</td>
<td>Reverse the result of the subtraction in Step 3. The reverse of 4995 is 5994.</td>
</tr>
<tr>
<td>5.</td>
<td>Add the results of Step 3 and Step 4. Black hole 10989 4995 + 5994 = 10989</td>
</tr>
</tbody>
</table>

**Question 1.** Suppose you apply the process to 1555, 2991, 6004, 7115, and 8225. Will they fall into black hole 10989? Test your answer by applying the process to the numbers.

**Question 2.** What do you notice about the numbers 1555, 2991, 6004, 7115, and 8225? Is there a pattern?

**Question 3.** Suppose you apply the process to 1545, 2971, 6024, 7315, and 8025. Will they fall into black hole 10989? Test your answer by applying the process to the numbers.
Your Turn  Apply the Black Hole process to 4181.

1. Start with 4181.  [4181 is a Fibonacci number.]
2. Reverse the starting number. Write it backwards.  The reverse of 4181 is _____.
3. Subtract the lesser number from the greater number.  _____ – _____ = _____
4. Reverse the result of the subtraction in Step 3.  The reverse of _____ is _____.
5. Add the results of Step 3 and Step 4.  _____ + _____ = _____

Answers

Start with 4181.  [4181 is a Fibonacci number.]

1. Reverse the starting number. Write it backwards.  The reverse of 4181 is 1814.
2. Subtract the lesser number from the greater number.  4181 – 1814 = 2367
3. Reverse the result of the subtraction in Step 3.  The reverse of 2367 is 7632.
4. Add the results of Steps 3 and 4. Black hole 9999.  2367 + 7632 = 9999

We applied the process to 1010, 1212, 2424, 3010, 4373, 5252, 6174, 7171, 8033, and 9595. They all fell into black hole 9999. Here is the process applied to 1212.

The Black Hole Process

1. Start with 1212.
2. Reverse the starting number. Write it backwards.  The reverse of 1212 is 2121.
3. Subtract the lesser number from the greater number.  2121 – 1212 = 909
4. Reverse the result of the subtraction in Step 3.  The reverse of 909 is 9090.
5. Add the results of Steps 3 and 4. Black hole 9999.  909 + 9090 = 9999

Question 1. Apply the process to 3232, 4343, 5454, 6565, and 7676. Do they fall into black hole 9999? What do you notice about the greatest – least result?

Question 2. Apply the process to 3131, 4242, 5353, 6464, 7575. Will they fall into black hole 10989? What do you notice about the greatest – least result?

Question 3. Apply the process to 3030, 4141, 5252, 6363, 7474. Will they fall into black hole 10989? What do you notice about the greatest – least result?
Internet Resources

Why does the Black Hole 1089 process work? You can find out at these Web sites:

  Why is it always 1089?  [http://www.mathsisfun.com/1089algebra.html]

  1089 and a Property of 3-digit Numbers
  [http://www.cut-the-knot.org/Curriculum/Arithmetic/S1089.shtml]

At Cut the Knot, scroll down to find links to sites for 4-digit numbers and 5-digit numbers.

Four Digits Magic Prediction

  •  [http://www.cut-the-knot.org/Curriculum/Arithmetic/FourDigitsPrediction.shtml]
  •  For 4-digit numbers, the process may lead to one of 990, 9999, 10890, 10899.

Five Digits Magic Prediction

  •  [http://www.cut-the-knot.org/Curriculum/Arithmetic/FiveDigitsPrediction.shtml]
  •  For 5-digit numbers, the process may lead to one of 109989, 109890, 99099, 10890.

At the Cut the Knot sites, a box displays a starting number and the results of applying the black hole process. Click on the starting number at the top of the stack to get a different starting number. Click, click, click – collect as many examples as you like, especially handy for 4-digit and 5-digit numbers.

Search for **number 1089** and you will find more Black Hole 1089 sites. Some of them say that the first and last digits of the starting number must differ by 2 or more so that that the difference of the starting number and its reverse is a 3-digit number, not a 2-digit number.

\[
\begin{align*}
321 - 123 &= 198 \\
512 - 215 &= 297 \\
602 - 206 &= 396 \\
823 - 328 &= 495
\end{align*}
\]

This restriction is unnecessary if you add leading zeros to the difference so that it becomes a number with the right number of digits.

\[
\begin{align*}
231 - 132 &= 99 \\
\text{Add a leading zero to get 099.} \\
099 + 990 &= 1089
\end{align*}
\]
The number 495 has many interesting properties, but the one we like best is that 495 is a mathemagical black hole using the process described here.

1. Start with a 3-digit natural number that has at least two different digits. [Can’t be 111, 222, 333, 444, 555, 666, 777, 888, or 999.]
2. Arrange the digits in order from greatest to least to get the greatest 3-digit number with these three digits.
3. Arrange the digits in order from least to greatest to get the least 3-digit number with these three digits.
4. Subtract the lesser number from the greater number to obtain a new number.
5. If the difference calculated in Step 4 is not equal to 495, use this new number in Step 2 and continue from there.
   If the difference calculated in Step 4 is equal to 495, stop. 495 is a mathemagical black hole.
   If you apply the process to 495, you will again get 495.

What skills do you need to apply this process? 1) Arrange the digits of a 3-digit number to get the greatest number using the three digits. 2) Arrange the digits of a 3-digit number to get the least number using the three digits. 3) Subtract the lesser number from the greater number. A calculator is appropriate technology for this step.

**Example 1.** Verify that 495 is a black hole. Apply the process to 495 and get 495.

1. 495 is a 3-digit natural number that has at least two different digits.
2. Arrange the digits of 495 in order from greatest to least: 954
3. Arrange the digits of 495 in order from least to greatest: 459
4. Subtract the lesser number from the greater number: 954 - 459 = 495.
5. The difference calculated in Step 4 is equal to 495, so 495 is a black hole.

If you apply the process to 495, the result is 495. Using this process, 495 is a mathemagical black hole.
We like to show black hole action in handy tables. The following table shows the steps in verifying that 495 is a black hole. The table heading $\text{max}$ is the greatest number using the digits of 495, and $\text{min}$ is the least number using the digits of 495.

<table>
<thead>
<tr>
<th>Start with 495</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>495</td>
</tr>
</tbody>
</table>

Yes, 495 is a black hole.

Example 2. The number 594 is a permutation of 495. Apply the process to 594.

There are six ways to arrange the three digits of a 3-digit number. These six ways are called **permutations**. The six permutations of 594 are:

- 459, 495, 549, 594, 945, and 954

1. 594 is a 3-digit number that has at least two different digits.
2. Arrange the digits of 594 in order from greatest to least: 954
3. Arrange the digits of 594 in order from least to greatest: 459
4. Subtract the lesser number from the greater number: $954 - 459 = 495$
5. The difference calculated in Step 4 is equal to 495. Black hole.

Applying the process to 594 results in black hole 495 in one application of steps 2 through 4. Here are the steps in a table.

<table>
<thead>
<tr>
<th>Start with 594</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>594</td>
</tr>
</tbody>
</table>

594 → 495 in one application of the process.

**Question.** 945 is a permutation of 495. How many applications of the process will it take to send 945 into black hole 495.

**Answer.** One application. Any permutation of 495 will become black hole 495 in one application of the process.
Your Turn Apply the Black Hole process to 945.

1. Start with 945. It is a permutation of 495.
2. Arrange the digits of 945 in order from greatest to least. _____
3. Arrange the digits of 945 in order from least to greatest. _____
4. Subtract the lesser number from the greater number. _____ – _____ = _____.
5. Did you get 495? If not, check your calculations.

Answers

1. Start with 945. Start with 945.
2. Arrange the digits of 945 in order from greatest to least. 954
3. Arrange the digits of 945 in order from least to greatest. 459
4. Subtract the lesser number from the greater number. 954 – 459 = 495
5. Black hole 495 in one application of the process. Black hole 495

Your Turn Apply the Black Hole process to 549.

1. Start with 549. It is a permutation of 495.
2. Arrange the digits of 549 in order from greatest to least. _____
3. Arrange the digits of 549 in order from least to greatest. _____
4. Subtract the lesser number from the greater number. _____ – _____ = _____.
5. Did you get 495? If not, check your calculations.

Answers

1. Start with 549. Start with 549.
2. Arrange the digits of 549 in order from greatest to least. 954
3. Arrange the digits of 549 in order from least to greatest. 459
4. Subtract the lesser number from the greater number. 954 – 459 = 495
5. Black hole 495 in one application of the process. Black hole 495
Example 3. Apply the process to 396. It will be gobbled into black hole 495 in two applications of the process.

1. 396 is a 3-digit number that has at least two different digits.
2. Arrange the digits of 396 in order from greatest to least: 963
3. Arrange the digits of 396 in order from least to greatest: 369
4. Subtract the lesser number from the greater number: 963 – 369 = 594
5. The difference calculated in Step 4 is not equal to 495. Use this number in Step 2 and continue from there.

2. Arrange the digits of 594 in order from greatest to least: 954
3. Arrange the digits of 594 in order from least to greatest: 459
4. Subtract the lesser number from the greater number: 954 – 459 = 495
5. The difference calculated in Step 4 is equal to 495. Black hole in two applications of the process. Here are the steps in a table.

<table>
<thead>
<tr>
<th>Start with 396</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>396</td>
</tr>
<tr>
<td>594</td>
</tr>
</tbody>
</table>

396 → 495 in two applications of the process.

Your Turn. Show that 258 is guzzled into black hole 495 in two applications of the process.

<table>
<thead>
<tr>
<th>Start with 258</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>258</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>258</td>
</tr>
<tr>
<td>594</td>
</tr>
</tbody>
</table>

258 → 495 in two applications? Yes
**Question.** 528 is a permutation of 258. How many applications of the process are required to zap 528 into black hole 495?

**Answer:** Two applications.

**Example 5.** The number 707 is dragged into black hole 495 in three applications of the process.

<table>
<thead>
<tr>
<th>number</th>
<th>max</th>
<th>min</th>
<th>max - min</th>
</tr>
</thead>
<tbody>
<tr>
<td>707</td>
<td>770</td>
<td>077</td>
<td>693</td>
</tr>
<tr>
<td>693</td>
<td>963</td>
<td>369</td>
<td>594</td>
</tr>
<tr>
<td>594</td>
<td>954</td>
<td>459</td>
<td>495</td>
</tr>
</tbody>
</table>

707 → 495 in three applications of the process.

**Your Turn** Show that 404 zooms into black hole 495 in three applications of the process.

<table>
<thead>
<tr>
<th>number</th>
<th>max</th>
<th>min</th>
<th>max - min</th>
</tr>
</thead>
<tbody>
<tr>
<td>404</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>____</td>
<td>____</td>
<td>____</td>
<td></td>
</tr>
<tr>
<td>____</td>
<td>____</td>
<td>____</td>
<td></td>
</tr>
</tbody>
</table>

707 → 495 in three applications? ____

<table>
<thead>
<tr>
<th>number</th>
<th>max</th>
<th>min</th>
<th>max - min</th>
</tr>
</thead>
<tbody>
<tr>
<td>404</td>
<td>440</td>
<td>044</td>
<td>396</td>
</tr>
<tr>
<td>396</td>
<td>963</td>
<td>369</td>
<td>594</td>
</tr>
<tr>
<td>594</td>
<td>954</td>
<td>459</td>
<td>495</td>
</tr>
</tbody>
</table>

404 → 495 in three applications? Yes
Example 6. The number 303 slips into black hole 495 in four applications of the process.

<table>
<thead>
<tr>
<th>number</th>
<th>max</th>
<th>min</th>
<th>max - min</th>
</tr>
</thead>
<tbody>
<tr>
<td>303</td>
<td>330</td>
<td>033</td>
<td>297</td>
</tr>
<tr>
<td>297</td>
<td>972</td>
<td>279</td>
<td>693</td>
</tr>
<tr>
<td>693</td>
<td>963</td>
<td>369</td>
<td>594</td>
</tr>
<tr>
<td>594</td>
<td>954</td>
<td>459</td>
<td>495</td>
</tr>
</tbody>
</table>

303 \rightarrow 495 in four applications of the process.

Example 7. The number 101 oozes into black hole 495 in six applications of the process.

The process requires 3-digit numbers at every step. If a step results in a 2-digit number, add a leading zero to make it a 3-digit number, as we do in this example.

<table>
<thead>
<tr>
<th>number</th>
<th>max</th>
<th>min</th>
<th>max - min</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>110</td>
<td>011</td>
<td>099*</td>
</tr>
<tr>
<td>099</td>
<td>990</td>
<td>099</td>
<td>891</td>
</tr>
<tr>
<td>891</td>
<td>981</td>
<td>189</td>
<td>792</td>
</tr>
<tr>
<td>792</td>
<td>972</td>
<td>279</td>
<td>693</td>
</tr>
<tr>
<td>693</td>
<td>963</td>
<td>369</td>
<td>594</td>
</tr>
<tr>
<td>594</td>
<td>954</td>
<td>459</td>
<td>495</td>
</tr>
</tbody>
</table>

101 \rightarrow 495 in six applications of the process.

* Add a leading zero to make a 3-digit number.
**Your Turn** Show that 232 zooms into black hole 495 in six applications of the process.

<table>
<thead>
<tr>
<th>number</th>
<th>max</th>
<th>min</th>
<th>max - min</th>
</tr>
</thead>
<tbody>
<tr>
<td>232</td>
<td>322</td>
<td>223</td>
<td>099*</td>
</tr>
<tr>
<td>099</td>
<td>990</td>
<td>099</td>
<td>891</td>
</tr>
<tr>
<td>891</td>
<td>981</td>
<td>189</td>
<td>792</td>
</tr>
<tr>
<td>792</td>
<td>972</td>
<td>279</td>
<td>693</td>
</tr>
<tr>
<td>693</td>
<td>963</td>
<td>369</td>
<td>594</td>
</tr>
<tr>
<td>594</td>
<td>954</td>
<td>459</td>
<td>495</td>
</tr>
</tbody>
</table>

404 → 495 in six applications? Yes

*Add a leading zero to make a 3-digit number.
Investigations

Have you noticed that certain numbers pop up frequently? Look back through the examples and exercises for permutations of numbers:

Permutations of 459: 459, 495, 549, 594, 945, and 954
Permutations of 369: 369, 396, 639, 693, 936, and 963
Permutations of 279: 279, 297, 729, 792, 927, and 972
Permutations of 189: 189, 198, 819, 891, 918, and 981.
Permutations of 099: 099, 909, and 990.

The number 495 is a black hole. Any permutation of 495 other than 495 itself is stuffed into black hole 495 in one application of the process. The table shows that 459, 594, and 954 go to black hole 495 in one application of the process (App #1).

<table>
<thead>
<tr>
<th>Number</th>
<th>App #1</th>
</tr>
</thead>
<tbody>
<tr>
<td>459</td>
<td>459 → 495</td>
</tr>
<tr>
<td>549</td>
<td>549 → 495</td>
</tr>
<tr>
<td>594</td>
<td>594 → 495</td>
</tr>
<tr>
<td>945</td>
<td>945 → 495</td>
</tr>
<tr>
<td>954</td>
<td>954 → 495</td>
</tr>
</tbody>
</table>

Any permutation of 396 will arrive at black hole 495 in two applications of the process. The table shows that he table shows that 369, 396, 639, 693, 936, and 963 go to black hole 495 in two applications of the process (App #1 and App #2). Notice that these numbers go to black hole 495 by way of 594.

<table>
<thead>
<tr>
<th>Number</th>
<th>App #1</th>
<th>App #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>369</td>
<td>369 → 594</td>
<td>594 → 495</td>
</tr>
<tr>
<td>396</td>
<td>396 → 594</td>
<td>594 → 495</td>
</tr>
<tr>
<td>639</td>
<td>639 → 594</td>
<td>594 → 495</td>
</tr>
<tr>
<td>693</td>
<td>693 → 594</td>
<td>594 → 495</td>
</tr>
<tr>
<td>936</td>
<td>936 → 594</td>
<td>594 → 495</td>
</tr>
<tr>
<td>963</td>
<td>963 → 594</td>
<td>594 → 495</td>
</tr>
</tbody>
</table>
**Your Turn** Any permutation of 279 will arrive at black hole 495 in three applications of the process. Complete the table showing that 279, 297, 729, 792, 927, and 972 go to black hole 495 in three applications of the process (App #1, App #2, and App #3).

<table>
<thead>
<tr>
<th>Number</th>
<th>App #1</th>
<th>App #2</th>
<th>App #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>279</td>
<td>279 → 693</td>
<td>693 → 594</td>
<td>594 → 495</td>
</tr>
<tr>
<td>297</td>
<td>297 → 693</td>
<td>693 → 594</td>
<td>594 → 495</td>
</tr>
<tr>
<td>729</td>
<td>729 → 693</td>
<td>693 → 594</td>
<td>594 → 495</td>
</tr>
<tr>
<td>792</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>927</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>972</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers**

<table>
<thead>
<tr>
<th>Number</th>
<th>App #1</th>
<th>App #2</th>
<th>App #3</th>
<th>App #4</th>
<th>App #5</th>
</tr>
</thead>
<tbody>
<tr>
<td>792</td>
<td>792 → 693</td>
<td>693 → 594</td>
<td>594 → 495</td>
<td></td>
<td></td>
</tr>
<tr>
<td>927</td>
<td>927 → 693</td>
<td>693 → 594</td>
<td>594 → 495</td>
<td></td>
<td></td>
</tr>
<tr>
<td>972</td>
<td>972 → 693</td>
<td>693 → 594</td>
<td>594 → 495</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We hope that you noticed: All permutations of 279 go to black hole 495 by way of 693 and 594.

We will do one more of these, the permutations 099, 909, and 990. Each of these permutations reaches black hole 495 in five applications of the process. They all go by way of 891, 792, 693, and 594.

<table>
<thead>
<tr>
<th>Number</th>
<th>App #1</th>
<th>App #2</th>
<th>App #3</th>
<th>App #4</th>
<th>App #5</th>
</tr>
</thead>
<tbody>
<tr>
<td>099</td>
<td>099 → 891</td>
<td>891 → 792</td>
<td>792 → 693</td>
<td>693 → 594</td>
<td>594 → 595</td>
</tr>
<tr>
<td>909</td>
<td>909 → 891</td>
<td>891 → 792</td>
<td>792 → 693</td>
<td>693 → 594</td>
<td>594 → 595</td>
</tr>
<tr>
<td>990</td>
<td>990 → 891</td>
<td>891 → 792</td>
<td>792 → 693</td>
<td>693 → 594</td>
<td>594 → 595</td>
</tr>
</tbody>
</table>
Investigate!

Investigate 100, 200, 300, 400, 500, 600, 700, 800, and 900.
Investigate 101, 202, 303, 404, 505, 606, 707, 808, and 909.
Investigate 123, 234, 345, 456, 567, 678, 789, permutations of these numbers.
Investigate 121, 232, 343, 454, 565, 676, 787, 898, and permutations of these numbers.
Investigate 135, 246, 357, 468, 579, and permutations of these numbers.
Investigate 501, 502, 503, 504, and permutations of these numbers.
Investigate – you choose the numbers.

A Class Project?

Three-digit numbers are the numbers from 100 to 999, inclusive. There are 900 3-digit numbers.
To be a candidate for black hole 495, a 3-digit number must have at least two different digits.
How many 3-digit numbers have at least two different digits?
How many 3-digit numbers go to black hole 495 in one application of the process? List them.
How many 3-digit numbers go to black hole 495 in two applications of the process? List them.
How many 3-digit numbers go to black hole 495 in three applications of the process? List them.
How many 3-digit numbers go to black hole 495 in four applications of the process? List them.
How many 3-digit numbers go to black hole 495 in five applications of the process? List them.
How many 3-digit numbers go to black hole 495 in six applications of the process? List them.
Et cetera, et cetera. Remember that all permutations of a number go to black hole 495 in the
same number of applications of the process. Once you find a number that goes to black hole 495
in n applications of the process, you can immediately put all permutations of that number in the n
applications box.

We now bid adieu to black hole 495 and move on to another mathemagical black hole.
Mathemagical Black Hole 6174

Black hole 6174 is the 4-digit equivalent of 3-digit black hole 495. The number 6174 is known as Kaprekar's constant. It is named for the Indian mathematician D. R. Kaprekar, who discovered the process that devours numbers into black hole 6174.

**The process:**

1. Start with a 4-digit natural number that has at least two different digits. [Can’t be 1111, 2222, 3333, 4444, 5555, 6666, 7777, 8888, or 9999.]
2. Arrange the digits in order from greatest to least to get the greatest 4-digit number with these four digits.
3. Arrange the digits in order from least to greatest to get the least 4-digit number with these four digits.
4. Subtract the lesser number from the greater number to obtain a new number.
5. If the difference calculated in Step 4 is not equal to 6174, use this new number in Step 2 and continue from there. If the difference calculated in Step 4 is equal to 6174, stop. 6174 is a mathemagical black hole. If you apply the process to 6174, you will again get 6174.

What skills do you need to apply this process? 1) Arrange the digits of a 4-digit number to get the greatest number using the four digits. 2) Arrange the digits of a 4-digit number to get the least number using the four digits. 3) Subtract the lesser number from the greater number. A calculator is appropriate technology for this step.

---

**Example 1.** Verify that 6174 is a black hole. Apply the process to 6174 and get 6174.

1. 6174 is a 4-digit natural number that has at least two different digits.
2. Arrange the digits of 6174 in order from greatest to least: 7641
3. Arrange the digits of 6174 in order from least to greatest: 1467
4. Subtract the lesser number from the greater number: 7641 – 1467 = 6174.
5. The difference calculated in Step 4 is equal to 6174, so 6174 is a black hole. If you apply the process to 6174, the result is 6174.
We like to show black hole action in tables. The following table shows the steps in verifying that 6174 is a black hole. The table heading max is the greatest number using the digits of 6174, and min is the least number using the digits of 6174.

<table>
<thead>
<tr>
<th>number</th>
<th>max</th>
<th>min</th>
<th>max – min</th>
</tr>
</thead>
<tbody>
<tr>
<td>6174</td>
<td>7641</td>
<td>1467</td>
<td>6174</td>
</tr>
</tbody>
</table>

Yes, 6174 is a black hole.

A permutation of the digits of a number is an arrangement of the digits into a particular order. For example, 12 and 21 are permutations of the 2-digit number with digits 1 and 2. The six permutations of the number 123 are 123, 132, 213, 231, 312, and 321. There are 24 permutations of the 4-digit number 1234, shown in the box below.

The 24 permutations of 1234 are:
1234, 1243, 1324, 1342, 1423, 1432
2134, 2143, 2314, 2341, 2413, 2431
3124, 3142, 3214, 3241, 3412, 3421
4123, 4132, 4213, 4231, 4312, 4321

Example 2. The number 4617 is a permutation of 6174. Apply the process to 4617.

1. 4617 is a 4-digit number that has at least two different digits.
2. Arrange the digits of 4617 in order from greatest to least: 7641
3. Arrange the digits of 4617 in order from least to greatest: 1467
4. Subtract the lesser number from the greater number: 7641 – 1467 = 6174
5. The difference calculated in Step 4 is equal to 6174. Applying the process to 4617 results in black hole 6174 in one application of steps 2 through 4.

<table>
<thead>
<tr>
<th>number</th>
<th>max</th>
<th>min</th>
<th>max – min</th>
</tr>
</thead>
<tbody>
<tr>
<td>4617</td>
<td>7641</td>
<td>1467</td>
<td>6174</td>
</tr>
</tbody>
</table>

4617 → 6174 in one application of the process.

Question. 1764 is a permutation of 6174. How many applications of the process will it take to send 1764 into black hole 6174?
**Answer.** One application. Any permutation of 6174 will become black hole 6174 in one application of the process.

**Example 3.** Start with 3087. Show the steps in a table.

<table>
<thead>
<tr>
<th>Start with 3087</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>3087</td>
</tr>
<tr>
<td>8352</td>
</tr>
</tbody>
</table>

3087 → 6174 in two applications of the process.
3087 → 8352 → 6174

**Example 4.** Start with 1211. Show the steps in a table.

<table>
<thead>
<tr>
<th>Start with 1211</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>1211</td>
</tr>
</tbody>
</table>

* If a number has fewer than 4 digits, add leading zeros to make a 4-digit number.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0999</td>
<td>9990</td>
<td>0999</td>
<td>8991</td>
</tr>
<tr>
<td>8991</td>
<td>9981</td>
<td>1899</td>
<td>8082</td>
</tr>
<tr>
<td>8082</td>
<td>8820</td>
<td>0288</td>
<td>8532</td>
</tr>
<tr>
<td>8532</td>
<td>8532</td>
<td>2358</td>
<td>6174</td>
</tr>
</tbody>
</table>

1211 → 6174 in five applications of the process.
1211 → 0999 → 8991 → 8082 → 8532 → 6174
Your Turn Do these activities before you peek at our Answers.

1. Zoom 7146 into black hole 6174 in one application of the process.

   \[
   \begin{array}{|c|c|c|c|}
   \hline
   \text{number} & \text{max} & \text{min} & \text{max} - \text{min} \\
   \hline
   7146 & 7641 & 1467 & 6174 \\
   \hline
   \end{array}
   \]

   7146 \rightarrow 6174 in one application of the process?

2. Nudge 2880 into black hole 6174 in two applications of the process.

   \[
   \begin{array}{|c|c|c|c|}
   \hline
   \text{number} & \text{max} & \text{min} & \text{max} - \text{min} \\
   \hline
   2880 & 8820 & 0288 & 8532 \\
   \hline
   \end{array}
   \]

   2880 \rightarrow 6174 in two applications of the process.

   \[
   \begin{array}{|c|c|c|c|}
   \hline
   \text{number} & \text{max} & \text{min} & \text{max} - \text{min} \\
   \hline
   8532 & 8532 & 2358 & 6174 \\
   \hline
   \end{array}
   \]

   2880 \rightarrow 8532 \rightarrow 6174

Answers

1. Start with 7146

   \[
   \begin{array}{|c|c|c|c|}
   \hline
   \text{number} & \text{max} & \text{min} & \text{max} - \text{min} \\
   \hline
   7146 & 7641 & 1467 & 6174 \\
   \hline
   \end{array}
   \]

   7146 \rightarrow 6174 in one application of the process.

2. Start with 2880

   \[
   \begin{array}{|c|c|c|c|}
   \hline
   \text{number} & \text{max} & \text{min} & \text{max} - \text{min} \\
   \hline
   2880 & 8820 & 0288 & 8532 \\
   \hline
   8532 & 8532 & 2358 & 6174 \\
   \hline
   \end{array}
   \]

   2880 \rightarrow 6174 in two applications of the process.

   2880 \rightarrow 8532 \rightarrow 6174
**Your Turn** Complete the process of cramming 3233 into black hole 6174 in five applications of the process. We have done the first step. The result of the first application is the 3-digit number 999, so we added a leading zero to make the 4-digit number 0999.

<table>
<thead>
<tr>
<th>number</th>
<th>max</th>
<th>min</th>
<th>max – min</th>
</tr>
</thead>
<tbody>
<tr>
<td>3233</td>
<td>3332</td>
<td>2333</td>
<td>0999*</td>
</tr>
</tbody>
</table>

*Add a leading zero to 999 to get 0999.

<table>
<thead>
<tr>
<th>number</th>
<th>max</th>
<th>min</th>
<th>max – min</th>
</tr>
</thead>
<tbody>
<tr>
<td>0999</td>
<td>9990</td>
<td>0999</td>
<td>8991</td>
</tr>
<tr>
<td>8991</td>
<td>9981</td>
<td>1899</td>
<td>8082</td>
</tr>
<tr>
<td>8082</td>
<td>8820</td>
<td>0288</td>
<td>8532</td>
</tr>
<tr>
<td>8532</td>
<td>8532</td>
<td>2358</td>
<td>6174</td>
</tr>
</tbody>
</table>

3233 → 6174 in five applications of the process.

3233 → 0999 → 8991 → 8082 → 8532 → 6174
Your Turn  Stuff 5545 into black hole 6174 in five applications of the process. If any result has fewer than four digits, add leading zeros to make a 4-digit number.

Start with 5545

<table>
<thead>
<tr>
<th>number</th>
<th>max</th>
<th>min</th>
<th>max – min</th>
</tr>
</thead>
<tbody>
<tr>
<td>5545</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5545 → 6174 in five applications of the process?
5545 → ______ → ______ → ______ → ______ → ______

Answers

Start with 5545

<table>
<thead>
<tr>
<th>number</th>
<th>max</th>
<th>min</th>
<th>max – min</th>
</tr>
</thead>
<tbody>
<tr>
<td>5545</td>
<td>5554</td>
<td>4555</td>
<td>0999*</td>
</tr>
<tr>
<td>0999</td>
<td>9990</td>
<td>0999</td>
<td>8991</td>
</tr>
<tr>
<td>8991</td>
<td>9981</td>
<td>1899</td>
<td>8082</td>
</tr>
<tr>
<td>8082</td>
<td>8820</td>
<td>0288</td>
<td>8532</td>
</tr>
<tr>
<td>8532</td>
<td>8532</td>
<td>2358</td>
<td>6174</td>
</tr>
</tbody>
</table>

*Add a leading zero to 999 to get 0999.

3233 → 6174 in five applications of the process.
5545 → 0999 → 8991 → 8082 → 8532 → 6174

Compare the sequences of results for 3233 and 5545:
3233 → 0999 → 8991 → 8082 → 8532 → 6174
5545 → 0999 → 8991 → 8082 → 8532 → 6174
Investigate

Permutations of 6174 other than 6174 itself are zapped into black hole 6174 in one application of the process. The 23 permutations not equal to 6174 are shown in this handy table.

| 1467, 1476, 1647, 1674, 1746, 1764 |
| 4167, 4176, 4617, 4671, 4716, 4761 |
| 6147, 6417, 6471, 6714, 6741 |
| 7146, 7164, 7416, 7461, 7614, 7641 |

As you have seen in the examples and Your Turns above, the numbers 3087 and 2880 plummet into black hole 6174 in two applications of the process. There are 24 permutations of 3087 and 24 permutations of 2880. However, some of the permutations of 2880 begin with zero, and 2880 has two digits equal to 8. The nine distinct (different) permutations of 2880 that do not begin with 0 are shown in the next table.

| 2088, 2808, 2880 |
| 8028, 8082, 8208, 8280, 8802, 8820 |

Are there more numbers that collapse into black hole 6174 in two applications of the process? Investigate!

The numbers 1211, 3233, and 5545 ooze into black hole 6174 in five applications of the process. Each of these numbers has three equal digits. That cuts way down on the number of different permutations. How many distinct permutations are there of each of these numbers?

- How many different permutations of 1211?
- How many different permutations of 3233?
- How many different permutations of 5545?

1. Verify that the process sucks 1234 into black hole 6174 in three applications. How many different permutations are there of 1234?

2. Verify that the process slurps 3996 into black hole 6174 in three applications. How many different permutations of 3996 are there?

3. While writing this section about black hole 6174, we didn't find any numbers that go to black hole 6174 in four applications of the process. A splendid opportunity for you to investigate. Are there numbers that reach black hole 6174 in exactly four applications of the process?

4. We didn't find any numbers that fall into black hole 6174 in five applications of the process or more than six applications. Another opportunity for you. Investigate!
Let’s generalize the process for $n$-digit numbers, where $n$ is any natural number that you fancy.

1. Start with an $n$-digit number, where $n$ is a natural number. If $n > 1$, then the selected number must have at least two different digits.
2. Arrange the digits in order from greatest to least to get the greatest $n$-digit number with the digits of the number.
3. Arrange the digits in order from least to greatest to get the least $n$-digit number with the digits of the number.
4. Subtract the lesser number from the greater number to obtain a new number.
5. If the new number calculated in Step 4 is not equal to a black hole number (495 for a 3-digit number or 6174 for a 4-digit number), use this new number in Step 2 and continue from there. If the new number calculated in Step 4 is equal to a black hole number (495 for a 3-digit number or 6174 for a 4-digit number), stop. Black hole. Using this process, 495 is a black hole for 3-digit numbers and 6174 is a black hole for 4-digit numbers.

What say $n = 1$? The 1-digit whole numbers are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Apply the process to 0, 1, and 2. You know that we like tables, so here are tables.

<table>
<thead>
<tr>
<th>number</th>
<th>max</th>
<th>min</th>
<th>max – min</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Zero (0) is a black hole.
Using this process, 0 is a black hole for 1-digit numbers, 495 is a black hole for 3-digit numbers, and 6174 is a black hole for 4-digit numbers.

1. Using this process, is there a 2-digit black hole? Investigate!
2. Using this process, is there a 5-digit black hole? Investigate!
3. And so on for 6-digit numbers, 7-digit numbers, and more-digit numbers. Investigate!

A Class Project?

Four-digit numbers are the numbers from 1000 to 9999, inclusive. There are 9000 4-digit numbers. How many of these numbers have at least two different digits?

How many 4-digit numbers go to black hole 6174 in one application of the process? List them.

How many 4-digit numbers go to black hole 6174 in two applications of the process? List them.

How many 4-digit numbers go to black hole 6174 in three applications of the process? List them.

How many 4-digit numbers go to black hole 6174 in four applications of the process? List them.

How many 4-digit numbers go to black hole 6174 in five applications of the process? List them.

Et cetera, et cetera. Remember that all permutations of a number go to black hole 6174 in the same number of applications of the process. Once you find a number that goes to black hole 6174 in \(n\) applications of the process, you can immediately put all permutations of that number in the same number of applications box.

Farewell Mathemagical Black Hole 6174. We move on to another mathemagical black hole.
Mathemagical Black Hole 153

The number 153 has the interesting property that it is equal to the sum of the cubes of its digits.

\[ 153 = 1^3 + 5^3 + 3^3 = 1 + 125 + 27 \]

Is this a rare thing, an unusual property of numbers? Or is it commonplace that a number is equal to the sum of the cubes of its digits? What 1-digit whole number is equal to the sum of the cubes of its digits? Are there any 2-digit numbers that are equal to the sums of the cubes of their digits? Investigate! Try 2-digit numbers 10, 11, 12, 13, and so on up to 99.

The 3-digit numbers 370, 371, and 407 are also equal to the sums of the cubes of their digits:

\[
\begin{align*}
370 &= 3^3 + 7^3 + 0^3 = 27 + 343 + 0 \\
371 &= 3^3 + 7^3 + 1^3 = 27 + 343 + 1 \\
407 &= 4^3 + 0^3 + 7^3 = 64 + 0 + 343
\end{align*}
\]

The number 153 has another property that is even more interesting. The number 153 is a mathemagical black hole using the following process:

1. Start with any 3-digit natural number that is divisible by 3.
2. Calculate the sum of the cubes of the digits of the starting number.
3. If the number calculated in Step 2 is not equal to 153, use it in Step 2 and continue. If the number calculated in Step 2 is equal to 153, stop. You are in black hole 153.

What skills do you need to apply this process? 1) Calculate the cubes of the three digits of a 3-digit number, and 2) add the three cubes of the digits. A calculator is appropriate technology for these tasks. The cubes of the decimal digits are shown in the box below.

<table>
<thead>
<tr>
<th>Cubes of the decimal digits:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0(^3) = 0, 1(^3) = 1, 2(^3) = 8, 3(^3) = 27, 4(^3) = 64, 5(^3) = 125</td>
</tr>
<tr>
<td>6(^3) = 216, 7(^3) = 343, 8(^3) = 512, 9(^3) = 729</td>
</tr>
</tbody>
</table>

**Example 1.** Verify that 153 is a black hole. Apply the process to 153 and get 153.

1. Start with 153. [153 is divisible by 3: 153 / 3 = 51]
2. Calculate the sum of the cubes of the digits of 153. \(1^3 + 5^3 + 3^3 = 1 + 125 + 27 = 153\).
3. The number calculated in Step 2 is equal to 153. Stop. Black hole 153.
Example 2. The number 315 is a permutation of 153. Apply the process to 315.

There are six ways to arrange the three digits of a 3-digit number. These six ways are called permutations. The six permutations of 153 are: 135, 153, 315, 351, 513, and 531

1. Start with 315. This number is divisible by 3: $315 / 3 = 105$.
2. Calculate the sum of the cubes of the digits of 315: $3^3 + 1^3 + 5^3 = 27 + 1 + 125 = 153$.
3. The number calculated in Step 2 is equal to 153. Stop. Black hole 153. The steps of the process are shown below in a table.

<table>
<thead>
<tr>
<th>Start with 315</th>
<th>number (315)</th>
<th>sum of cubes of digits ($27 + 1 + 125$)</th>
<th>new number (153)</th>
</tr>
</thead>
<tbody>
<tr>
<td>315</td>
<td>27 + 1 + 125</td>
<td>153</td>
<td></td>
</tr>
</tbody>
</table>

315 → 153 in one application of the process

One application of the process stuffed 315 into black hole 153. The numbers 315 and 153 have the same digits, so the sums of the cubes of their digits will be the same. Do you think that one application of this process will slurp 135 or 351 or 513 or 531 into black hole 153?

Example 3. Apply the process to 108. Show the steps of the process in a table.

<table>
<thead>
<tr>
<th>Start with 108</th>
<th>number (108)</th>
<th>sum of cubes of digits ($1 + 0 + 512$)</th>
<th>new number (513)</th>
</tr>
</thead>
<tbody>
<tr>
<td>108</td>
<td>1 + 0 + 512</td>
<td>513</td>
<td></td>
</tr>
<tr>
<td>513</td>
<td>125 + 1 + 27</td>
<td>153</td>
<td></td>
</tr>
</tbody>
</table>

108 → 153 in two applications of the process.

108 → 513 → 153
Example 4. Apply the process to 792. Show the steps of the process in a table. The first new number is 1080, a 4-digit number. That’s OK. Calculate the sum of the cubes of the four digits of 1080.

<table>
<thead>
<tr>
<th>Start with 792</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>792</td>
</tr>
<tr>
<td>1080</td>
</tr>
<tr>
<td>513</td>
</tr>
</tbody>
</table>

792 → 1080 → 513 → 153

Did you notice that 513 appeared in the second application of the process? 513 is a permutation of 153.

Example 5. Apply the process to 111. Show the steps in a table. The first new number is 3, a 1-digit number, and the second new number is 27, a 2-digit number. That’s OK. Calculate the sum of the cubes of the one digit of 3 and the two digits of 27.

<table>
<thead>
<tr>
<th>Start with 111</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>111</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>27</td>
</tr>
<tr>
<td>351</td>
</tr>
</tbody>
</table>

111 → 153 in four applications of the process.
111 → 3 → 27 → 351 → 153

Did you notice that the number 351 appeared in the third application of the process? 351 is a permutation of 153.
**Your Turn** Do these activities *before* you peek at our *Answers* down yonder.

1. The number 351 is a permutation of 153. Show that 351 is swallowed into black hole 153 in one application of the process.

<table>
<thead>
<tr>
<th></th>
<th>number</th>
<th>sum of cubes of digits</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>351</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

351 → 153 in one application of the process?

2. Show that 810 is gulped into black hole 153 in two applications of the process.

<table>
<thead>
<tr>
<th></th>
<th>number</th>
<th>sum of cubes of digits</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>810</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

810 → 153 in two applications of the process?

810 → _____ → _____

3. Show that 279 is slurped into black hole 153 in three applications of the process.

<table>
<thead>
<tr>
<th></th>
<th>number</th>
<th>sum of cubes of digits</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>279</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

279 → 153 in three applications of the process?

279 → _____ → _____ → _____
### Answers

#### 1. Start with 351

<table>
<thead>
<tr>
<th>number</th>
<th>sum of cubes of digits</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>351</td>
<td>$27 + 125 + 1$</td>
<td>153</td>
</tr>
</tbody>
</table>

$351 \rightarrow 153$ in one application of the process.

#### 2. Start with 810

<table>
<thead>
<tr>
<th>number</th>
<th>sum of cubes of digits</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>810</td>
<td>$512 + 1 + 0$</td>
<td>513</td>
</tr>
<tr>
<td>513</td>
<td>$125 + 1 + 27$</td>
<td>153</td>
</tr>
</tbody>
</table>

$810 \rightarrow 153$ in two applications of the process.  
$810 \rightarrow 513 \rightarrow 153$

#### 3. Start with 279

<table>
<thead>
<tr>
<th>number</th>
<th>sum of cubes of digits</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>279</td>
<td>$8 + 343 + 729$</td>
<td>1080</td>
</tr>
<tr>
<td>1080</td>
<td>$1 + 0 + 512 + 0$</td>
<td>513</td>
</tr>
<tr>
<td>513</td>
<td>$125 + 1 + 27$</td>
<td>153</td>
</tr>
</tbody>
</table>

$279 \rightarrow 153$ in three applications of the process.  
$279 \rightarrow 1080 \rightarrow 513 \rightarrow 153$

**Did you notice** that, in Exercises 2 and 3, 513 appeared in the step just before the appearance of black hole 153? Guess what! Yep, 513 is a permutation of 153.
Your Turn Show that 222 plunges into black hole 153 in four applications of the process.

<table>
<thead>
<tr>
<th>Start with 222</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>222</td>
</tr>
<tr>
<td>24</td>
</tr>
<tr>
<td>72</td>
</tr>
<tr>
<td>351</td>
</tr>
</tbody>
</table>

222 → 153 in four applications of the process?

222 → _____ → _____ → _____ → _____

Answers

<table>
<thead>
<tr>
<th>Start with 222</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>222</td>
</tr>
<tr>
<td>24</td>
</tr>
<tr>
<td>72</td>
</tr>
<tr>
<td>351</td>
</tr>
</tbody>
</table>

222 → 153 in four applications of the process.

222 → 24 → 72 → 351 → 153

Question. What number appeared as the result of the third application of the process? Use this number to help complete the following sentence:

The number _____ is a ________________ of 153.
**Investigate**

**Conjecture:** Using the sum of cubes of digits process, every 3-digit number divisible by 3 ends up in black hole 153 after a finite number of applications of the process.

For examples and Your Turn activities, we chose starting numbers that would fall into black hole 153 in a few applications of the process. Here are more examples. We use arrows to show the starting number, each new number, and black hole 153 at the end of the process.

Starting number 333 requires three applications of the process.

\[ 333 \rightarrow 81 \rightarrow 513 \rightarrow 153 \]

Starting number 303 requires six applications of the process.

\[ 303 \rightarrow 54 \rightarrow 189 \rightarrow 1242 \rightarrow 81 \rightarrow 513 \rightarrow 153 \]

Starting number 123 requires seven applications of the process.

\[ 123 \rightarrow 36 \rightarrow 243 \rightarrow 99 \rightarrow 1458 \rightarrow 702 \rightarrow 351 \rightarrow 153 \]

Starting number 105 requires 10 applications of the process.

\[ 105 \rightarrow 126 \rightarrow 225 \rightarrow 141 \rightarrow 66 \rightarrow 432 \rightarrow 99 \rightarrow 1458 \rightarrow 702 \rightarrow 351 \rightarrow 153 \]

We hope that you might be a bit skeptical about the above and verify our calculations, or find any mistakes that we maid.

Three-digit numbers are the numbers 100, 101, 102, 103, and so on up to 997, 998, and 999. There are 900 three-digit numbers.

**Question.** How many 3-digit numbers are divisible by 3? Here are the first 33 three-digit numbers that are divisible by 3.


**Question.** How many 3-digit numbers divisible by 3 fall into black hole 153 in one application of the process? List these numbers.

**Question.** How many 3-digit numbers divisible by 3 fall into black hole 153 in two applications of the process? List these numbers.

**Question.** How many 3-digit numbers divisible by 3 fall into black hole 153 in three applications of the process? List these numbers.

**Et cetera, et cetera.** Continue for four applications of the process, or more applications of the process. Remember, all permutations of a number require the same number of applications of the process to fall into black hole 153. [153 requires no applications of the process, but permutations 135, 315, 351, 513, and 531 each require one application.]
**Question.** What 3-digit numbers divisible by 3 require the largest number of applications of the process to tumble into black hole 153? List these numbers.

**Question.** The number 370 and its permutations 307, 703, and 730 do not fall into black hole 153. Apply the process to these numbers and see what happens. Why don't they fall into black hole 153?

**Question.** The number 371 and its permutations 137, 173, 317, 713, and 731 do not fall into black hole 153. Apply the process to these numbers and see what happens. Why don't they fall into black hole 153?

**Question.** The number 407 and its permutations 470, 704, and 740 do not fall into black hole 153. Apply the process to these numbers and see what happens. Why don't they fall into black hole 153?

**Classroom Project**

Sort the numbers from 100 to 999 into categories such as these:

- Do not fall into black hole 153.
- Fall into black hole 153 in one application of the process.
- Fall into black hole 153 in two applications of the process.
- Fall into black hole 153 in three applications of the process.
- Fall into black hole 153 in four applications of the process.
- And so on.

You can form teams of students and assign each team part of the task. Form nine teams and have each team investigate one/ninth of the 3-digit numbers divisible by 3. Numbers that are divisible by 3 are also called **multiples of 3**. Team 1 investigates multiples of 3 from 100 through 199, Team 2 takes on multiples of 3 from 200 through 299, Team 3 studies multiples of 3 from 300 through 399, and so on.

Each team has only 33 or 34 numbers to investigate. Combine the data and list the numbers by number of applications: 1 application, 2 applications, 3 applications, and so on. Also make a graph that shows the data visually.

**Investigate!**
Stardate 2012-02-01. Bob has been tutoring a remarkable student for three years. We call him SpockData because he loves Star Trek and loves logic. We began with Algebra 1 two years ago, then Geometry last year, and now Algebra 2 (2011-2012 school year). He has blossomed. This year he is pulling an A in Algebra 2, so in our tutoring gigs, we do alternative math (we call it Bob & George math) that is sometimes a bit afield of the usual high school math. One day we showed SpockData the square root black hole.

The square root process: Start with any real number. Calculate the square root of the starting number. Calculate the square root of the result. Then calculate the square root of that result. Keep on square rooting – calculate the square root of each result.

For example, start with 100.

\[
\sqrt{100} = 10 \\
\sqrt{10} = 3.16227766 \\
\sqrt{3.16227766} = 1.77827941 \\
\sqrt{1.77827941} = 1.333521432
\]

Whoa! Too much work.

Entering all those digits for each square root calculation is tedious, time consuming, and prone to error. The TI-84 Graphing Calculator to the rescue. The TI-84 makes it absurdly easy to take the square root of the previous answer.

If you know how to use the TI-84, you know that you can use [2nd]√ [2nd] ANS to quickly calculate the square root of the previous answer.

After you do this once, you need press only the Enter key to get each additional square root. Each time you press Enter, you get the next square root in the sequence. Abracadabra!

If you don't know how to use the TI-84 to do this square root task, use any calculator that can calculate square roots.
Using the TI-84, and starting with 100, SpockData quickly ran off the first 10 square roots. We show them here as they appear in the TI-84 display and also rounded to four decimal places.

<table>
<thead>
<tr>
<th>Square roots ala TI-84</th>
<th>Rounded to 4 decimal places</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>3.16227766</td>
<td>3.1623</td>
</tr>
<tr>
<td>1.77827941</td>
<td>1.7783</td>
</tr>
<tr>
<td>1.333521432</td>
<td>1.3335</td>
</tr>
<tr>
<td>1.154781985</td>
<td>1.1548</td>
</tr>
<tr>
<td>1.074607828</td>
<td>1.0746</td>
</tr>
<tr>
<td>1.036632928</td>
<td>1.0366</td>
</tr>
<tr>
<td>1.018151722</td>
<td>1.0182</td>
</tr>
<tr>
<td>1.009035045</td>
<td>1.0090</td>
</tr>
<tr>
<td>1.004507364</td>
<td>1.0045</td>
</tr>
</tbody>
</table>

Bob asked SpockData to make observations about what is happening. SD's observations:

- The numbers are decreasing.
- The numbers are all greater than 1.
- The numbers are getting closer to 1. They seem to be "homing in" on 1.
- The decimal to the right of the decimal point goes down by a factor of about two each time.

Abracadabra! Alakazam! Bob had not noticed that last observation. Score math maturity points for SpockData.

Will this continue? Here are the next five numbers.

| 1.002251148 | 1.0023 |
| 1.001124941 | 1.0011 |
| 1.000562313 | 1.0006 |
| 1.000281117 | 1.0003 |
| 1.000140549 | 1.0001 |
SpockData kept pressing the Enter key and eventually the TI-84’s display showed exactly 1. They discussed this and agreed that that happened because the TI-84 rounds numbers to 10 digits in its display. A calculator that could display an unlimited number of digits would never reach 1, but get closer to 1 after every square root calculation. The black hole is 1, but you never get there – you can only get closer and closer and closer.

SpockData had noticed that the decimal part of the number (the part to the right of the decimal point) seemed to decrease by a factor of approximately 2 each time. He wanted to investigate this, so he calculated the ratios. Here are some of his calculations, beginning with the decimal part of 1.77827941. The calculations are rounded to five decimal places.

\[
\frac{0.77827941}{0.333521432} = 2.33352
\]
\[
\frac{0.333521432}{0.154781985} = 2.07461
\]
\[
\frac{0.154781985}{0.074607828} = 2.03663
\]
\[
\frac{0.074607828}{0.036632928} = 2.01815
\]
\[
\frac{0.036632928}{0.018151722} = 2.01815
\]
\[
\frac{0.018151722}{0.009035045} = 2.00904
\]
\[
\frac{0.009035045}{0.004507364} = 2.00451
\]
\[
\frac{0.004507364}{0.002251148} = 2.00225
\]
\[
\frac{0.002251148}{0.001124941} = 2.00113
\]
\[
\frac{0.001124941}{0.000562313} = 2.00056
\]

Aha! The ratio is not only approximately equal to 2, it also seems to be approaching 2. The decimal part of the number again seems to be decreasing by a factor of approximately 2 each time. SpockData has quite a bit of evidence (not proof) to support the following conjectures.

1. For a starting number greater than 1, and a calculator with an unlimited number of digits, each square root is less than the preceding one.

2. If he uses a calculator with an unlimited number of digits, all of the square roots will be greater than 1.

3. The square roots approach 1, but never reach 1. Each square root is closer to 1 than was the preceding square root.

4. The decimal parts of the numbers decrease by a factor of approximately 2 each time. The ratio of successive decimal parts approaches 2.

4. The black hole for this square root process is 1, but is never attained. Using a calculator with an unlimited number of digits, you will get closer and closer to 1, but never reach 1.
Your Turn. Investigate.

SpockData and Bob applied the square root process to a starting number greater than 1. What happens if the starting number is less than 1, but greater than 0? For example, what happens if the starting number is 0.5? Assume that you are using an imaginary calculator that can calculate with an unlimited number of digits.

1. Will each square root be less than the preceding square root? Or will it be greater than the preceding one?

2. The starting number 0.5 is less than 1. Will all of the square roots be less than 1?

3. Will the numbers approach 1? Will each square root be closer to 1 than the preceding square root?

Investigate!
Mathemagical Black Holes of the Cyclic Kind

Now we delve into processes that slurp some numbers into a single black hole, but whirl other numbers into a cyclic black hole. A cyclic black hole is a sequence of two or more different numbers that repeat again and again and again, endlessly, never ending.

The Sum of Squares of Digits Process

Start with any natural number and calculate the sum of the squares of the digits of the number. Some numbers fall into black hole 1; other numbers end up in a cyclic black hole.

Squares of the decimal digits:
- $0^2 = 0$, $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$
- $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$

**Example 1.** Show that 1 is a black hole using the sum of squares of digits process.

<table>
<thead>
<tr>
<th>number</th>
<th>sum of squares</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

1 is a black hole using the process. $1 \rightarrow 1$

**Example 2.** Show that 10 tumbles into black hole 1 in one application of the sum of squares of digits process.

<table>
<thead>
<tr>
<th>number</th>
<th>sum of squares</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$1 + 0$</td>
<td>1</td>
</tr>
</tbody>
</table>

10 $\rightarrow$ 1 in one application of the sum of squares of digits process.

**Question.** In how many applications of the sum of squares of digits process will 100 fall into black hole 1?
Example 3. Show that 68 reaches black hole 1 in two applications of the sum of squares of digits process.

<table>
<thead>
<tr>
<th>Start with 68</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>68</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

68 → 1 in two applications of the process.
68 → 100 → 1

Your Turn Show that 13 is gobbled into black hole 1 in two applications of the sum of squares of digits process.

<table>
<thead>
<tr>
<th>Start with 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>13</td>
</tr>
</tbody>
</table>

13 → 1 in two applications of the process?
13 → ____ → ____

Answers

<table>
<thead>
<tr>
<th>Start with 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

13 → 1 in two applications of the process.
13 → 10 → 1
Your Turn Show that 28 is inhaled into black hole 1 in three applications of the sum of squares of digits process.

<table>
<thead>
<tr>
<th>number</th>
<th>sum of squares</th>
<th>new number</th>
<th>28 → 1 in three applications of the process?</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td></td>
<td></td>
<td>28 → ____ → ____ → ____</td>
</tr>
</tbody>
</table>

Answers

<table>
<thead>
<tr>
<th>number</th>
<th>sum of squares</th>
<th>new number</th>
<th>28 → 1 in three applications of the process.</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>4 + 64</td>
<td>68</td>
<td>28 → 100 → 10 → 1</td>
</tr>
<tr>
<td>68</td>
<td>36 + 64</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1 + 0 + 0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Question. 82 is a permutation of 28. Will 82 be inhaled into black hole 1 in three applications of the sum of squares of digits process?

If two numbers have exactly the same digits, then the sum of the squares of the digits of the two numbers will be the same.

13 and 31 have the same digits. \(1^2 + 3^2 = 1 + 9 = 10\) \(3^2 + 1^2 = 9 + 1 = 10\) 
28 and 82 have the same digits. \(2^2 + 8^2 = 4 + 64 = 68\) \(8^2 + 2^2 = 64 + 4 = 68\) 
68 and 86 have the same digits. \(6^2 + 8^2 = 36 + 64 = 100\) \(8^2 + 6^2 = 64 + 36 = 100\)
Your Turn Show that 44 tumbles into black hole 1 in four applications of the sum of squares of digits process.

<table>
<thead>
<tr>
<th>number</th>
<th>sum of squares</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>44</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

44 → 1 in four applications of the process?

44 → ____ → ____ → ____ → ____

Answers

<table>
<thead>
<tr>
<th>number</th>
<th>sum of squares</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>44</td>
<td>16 + 16</td>
<td>32</td>
</tr>
<tr>
<td>32</td>
<td>9 + 4</td>
<td>13</td>
</tr>
<tr>
<td>13</td>
<td>1 + 9</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>1 + 0</td>
<td>1</td>
</tr>
</tbody>
</table>

44 → 1 in four applications of the process.

44 → 32 → 13 → 10 → 1

Palindromes. 44 is a palindrome. The reverse of 44 is 44. Read it from left or right or right to left – same number. We wonder. Do other 2-digit palindromes fall into black hole 1? The other 2-digit palindromes are 11, 22, 33, 55, 66, 77, 88, and 99.
**Your Turn** Show that 70 oozes into black hole 1 in five applications of the sum of squares of digits process.

<table>
<thead>
<tr>
<th>number</th>
<th>sum of squares</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49</td>
<td>49 + 0</td>
<td>49</td>
</tr>
<tr>
<td>97</td>
<td>81 + 49</td>
<td>130</td>
</tr>
<tr>
<td>130</td>
<td>1 + 9 + 0</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>1 + 0</td>
<td>1</td>
</tr>
</tbody>
</table>

70 → 1 in five applications of the process.

The sequence of numbers leading to black hole 1 is: 70 → 49 → 97 → 130 → 10 → 1

Peruse this table and you can see that 70 falls into black hole 1 in five applications of the sum of squares of digits process, 49 falls into black hole 1 in four applications of the process, 97 in three applications, 130 in two applications, and 10 in one application.
Your Turn: Investigate

We have applied the sum of squares of digits process to 2-digit numbers, found numbers that fall into black hole 1 in one to five applications of the process, and recorded some of our findings in the handy table below. For a few numbers, we put an x under the number of apps (applications) required to plunge the number into black hole 1. We hope that you will complete the table.

<table>
<thead>
<tr>
<th>Number</th>
<th>1 app</th>
<th>2 apps</th>
<th>3 apps</th>
<th>4 apps</th>
<th>5 apps</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>44</td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>68</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>79</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>82</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>86</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>91</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Squares of the decimal digits:
\[0^2 = 0, \ 1^2 = 1, \ 2^2 = 4, \ 3^2 = 9, \ 4^2 = 16\]
\[5^2 = 25, \ 6^2 = 36, \ 7^2 = 49, \ 8^2 = 64, \ 9^2 = 81\]
Cyclic Black Holes

As perhaps you suspect, we have been choosing starting numbers that fall into black hole 1. Now we will choose starting numbers that immediately become cyclic black holes using the sum of squares of digits process.

Example 4. The number 4 is the beginning of a cyclic black hole using the sum of squares of digits process. The cycle begins with 4. After 8 applications of the process, 4 appears again and the cycle repeats. The cycle contains the numbers 4, 16, 37, 58, 89, 145, 42, and 20. There are eight different numbers in the cycle, so the cycle length is 8.

<table>
<thead>
<tr>
<th>number</th>
<th>sum of squares</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>16</td>
<td>$1 + 36$</td>
<td>37</td>
</tr>
<tr>
<td>37</td>
<td>$9 + 49$</td>
<td>58</td>
</tr>
<tr>
<td>58</td>
<td>$25 + 64$</td>
<td>89</td>
</tr>
<tr>
<td>89</td>
<td>$64 + 81$</td>
<td>145</td>
</tr>
<tr>
<td>145</td>
<td>$1 + 16 + 25$</td>
<td>42</td>
</tr>
<tr>
<td>42</td>
<td>$16 + 4$</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>$4 + 0$</td>
<td>4</td>
</tr>
</tbody>
</table>

4 begins a cycle of length 8.
Cycle numbers: 4, 16, 37, 58, 89, 145, 42, 20
Your Turn. The number 16 begins a cycle. With what number does the cycle end before 16 appears again to restart the cycle? What are the numbers in the cycle? What is the cycle length?

<table>
<thead>
<tr>
<th>Start with 16</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>1 + 36</td>
<td>37</td>
</tr>
<tr>
<td>37</td>
<td>9 + 49</td>
<td>58</td>
</tr>
<tr>
<td>58</td>
<td>25 + 64</td>
<td>89</td>
</tr>
<tr>
<td>89</td>
<td>64 + 81</td>
<td>145</td>
</tr>
<tr>
<td>145</td>
<td>1 + 16 + 25</td>
<td>42</td>
</tr>
<tr>
<td>42</td>
<td>16 + 4</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>4 + 0</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

Cycle numbers:

The cycle begins with 16, ends with 4, and then repeats. The cycle length is 8; there are 8 numbers in the cycle. Below are three cycles of the cyclic black hole that begins with 16:

16, 37, 58, 89, 145, 42, 20, 4
Your Turn. The number 37 begins a cycle. With what number does the cycle end before 37 appears again to restart the cycle? What are the numbers in the cycle? What is the cycle length?

Start with 37

<table>
<thead>
<tr>
<th>Start with 37</th>
</tr>
</thead>
<tbody>
<tr>
<td>37</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Cycle numbers:

____, ____ , ____ , ____, ____, ____, ____, ____

Answers

Start with 16

<table>
<thead>
<tr>
<th>Start with 16</th>
</tr>
</thead>
<tbody>
<tr>
<td>37</td>
</tr>
<tr>
<td>9 + 49</td>
</tr>
<tr>
<td>58</td>
</tr>
<tr>
<td>25 + 64</td>
</tr>
<tr>
<td>89</td>
</tr>
<tr>
<td>64 + 81</td>
</tr>
<tr>
<td>145</td>
</tr>
<tr>
<td>1 + 16 + 25</td>
</tr>
<tr>
<td>42</td>
</tr>
<tr>
<td>16 + 2</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>4 + 0</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>1 + 36</td>
</tr>
<tr>
<td>37</td>
</tr>
</tbody>
</table>

Cycle numbers:

37, 58, 89, 145, 42, 20, 4, 16

The cycle begins with 37 and ends with 16. Then the cycle restarts with 37 and repeats. The cycle length is 8; there are 8 numbers in the cycle. Below are three cycles of the cyclic black hole that begins with 37:

Each of the numbers 4, 16, 37, 58, 89, 145, 42, and 20 begins a cyclic black hole of length 8. For each of these numbers, the first two cycles are shown here:

Start with 4: \(4, 16, 37, 58, 89, 145, 42, 20, 4, 16, 37, 58, 89, 145, 42, 20, \) et cetera

Start with 16: \(16, 37, 58, 89, 145, 42, 20, 4, 16, 37, 58, 89, 145, 42, 20, 4, \) et cetera

Start with 37: \(37, 58, 89, 145, 42, 20, 4, 16, 37, 58, 89, 145, 42, 20, 4, 16, \) et cetera

Start with 58: \(58, 89, 145, 42, 20, 4, 16, 37, 58, 89, 145, 42, 20, 4, 16, 37, \) et cetera

Start with 89: \(89, 145, 42, 20, 4, 16, 37, 58, 89, 145, 42, 20, 4, 16, 37, 58, \) et cetera

Start with 145: \(145, 42, 20, 4, 16, 37, 58, 89, 145, 42, 20, 4, 16, 37, 58, 89, \) et cetera

Start with 42: \(42, 20, 4, 16, 37, 58, 89, 145, 42, 20, 4, 16, 37, 58, 89, 145, \) et cetera

Start with 20: \(20, 4, 16, 37, 58, 89, 145, 42, 20, 4, 16, 37, 58, 89, 145, 42, \) et cetera

Each cyclic black hole has a starting number and an ending number. Below we have used green type for the starting number and red for the ending number of a cycle.

Start with 4: The cycle begins with 4 and ends with 20: \(4, 16, 37, 58, 89, 145, 42, 20, 4, 16, 37, 58, 89, 145, 42, 20, \) et cetera

Start with 16: The cycle begins with 16 and ends with 4: \(16, 37, 58, 89, 145, 42, 20, 4, 16, 37, 58, 89, 145, 42, 20, 4, \) et cetera

Start with 37: The cycle begins with 37 and ends with 16: \(37, 58, 89, 145, 42, 20, 4, 16, 37, 58, 89, 145, 42, 20, 4, 16, \) et cetera

Start with 58: The cycle begins with 58 and ends with 37: \(58, 89, 145, 42, 20, 4, 16, 37, 58, 89, 145, 42, 20, 4, 16, 37, \) et cetera

Start with 89: The cycle begins with 89 and ends with 58: \(89, 145, 42, 20, 4, 16, 37, 58, 89, 145, 42, 20, 4, 16, 37, 58, \) et cetera

Start with 145: The cycle begins with 145 and ends with 89: \(145, 42, 20, 4, 16, 37, 58, 89, 145, 42, 20, 4, 16, 37, 58, 89, \) et cetera

Start with 42: The cycle begins with 42 and ends with 145: \(42, 20, 4, 16, 37, 58, 89, 145, 42, 20, 4, 16, 37, 58, 89, 145, \) et cetera

Start with 20: The cycle begins with 20 and ends with 42: \(20, 4, 16, 37, 58, 89, 145, 42, 20, 4, 16, 37, 58, 89, 145, 42, \) et cetera

83
You have seen examples of numbers that fall into black hole 1, and examples of numbers that begin a cyclic black hole. Strange – all the cyclic black hole examples have cycle length equal to 8, and the same eight numbers appear in every cycle.

There is a third type of happening. There are numbers that are not part of a cycle and do not fall into black hole 1. These numbers reach a cyclic black hole number after one or more applications of the sum of squares of digits process, and then the cycle takes over. The starting number is not seen again.

**Example 5.** The starting number 2 reaches cyclic black hole starting number 4 in one application of the process, the cycle takes over, and 2 does not appear again.

<table>
<thead>
<tr>
<th>number</th>
<th>sum of squares</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>16</td>
<td>1 + 36</td>
<td>37</td>
</tr>
<tr>
<td>37</td>
<td>9 + 49</td>
<td>58</td>
</tr>
<tr>
<td>58</td>
<td>25 + 64</td>
<td>89</td>
</tr>
<tr>
<td>89</td>
<td>64 + 81</td>
<td>145</td>
</tr>
<tr>
<td>145</td>
<td>1 + 16 + 25</td>
<td>42</td>
</tr>
<tr>
<td>42</td>
<td>16 + 4</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>4 + 0</td>
<td>4</td>
</tr>
</tbody>
</table>

2 reaches cyclic black hole number 4 in one application of the process.

Cycle numbers: 4, 16, 37, 58, 89, 145, 42, 20

Cycle length = 8.

Here is the starting number 2 and first cycle of the cyclic black hole that begins with 4:

2 → 4 → 16 → 37 → 58 → 89 → 145 → 42 → 20 → et cetera, et cetera
Your Turn. Show that 38 arrives at a cyclic black hole number in two applications of the sum of squares of digits process. What is the first cyclic black hole number to appear? What are the numbers in the cyclic black hole? What is the cycle length of the cyclic black hole?

<table>
<thead>
<tr>
<th>Start with 38</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>38</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
</tr>
</tbody>
</table>

Does 38 reach a cyclic black hole number in two applications of the process?

Answers

The number 38 arrives at cyclic black hole number 58 in two applications of the sum of squares of digits process. Here is the sequence of numbers, including the first cycle of the cyclic black hole that begins with 58:

\[
38 \rightarrow 73 \rightarrow 58 \rightarrow 89 \rightarrow 145 \rightarrow 42 \rightarrow 20 \rightarrow 4 \rightarrow 16 \rightarrow 37 \rightarrow 58 \text{ et cetera}
\]

The first cyclic black hole number is 58. The numbers in the cycle are 58, 89, 145, 42, 20, 4, 16, and 37. The cycle length is 8.
**Your Turn.** Show that 105 arrives at a cyclic black hole number in three applications of the sum of squares of digits process. What is the first cyclic black hole number to appear? What are the numbers in the cyclic black hole? What is the cycle length of the cyclic black hole?

<table>
<thead>
<tr>
<th>number</th>
<th>sum of squares</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td>105</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Does 105 reach a cyclic black hole number in three applications of the process?

**Answers**

The number 105 arrives at cyclic black hole number 16 in three applications of the sum of squares of digits process. Here is the sequence of numbers, including the first cycle of the cyclic black hole that begins with 58:

$$105 \rightarrow 26 \rightarrow 40 \rightarrow 16 \rightarrow 37 \rightarrow 58 \rightarrow 89 \rightarrow 145 \rightarrow 42 \rightarrow 20 \rightarrow 4 \rightarrow 16 \rightarrow \text{et cetera}$$

The first cyclic black hole number is 16. The numbers in the cycle are 16, 37, 58, 89, 145, 42, 20, and 4. The cycle length is 8.
Sum of Squares of Digits Conjectures – Investigate!

Applying the sum of squares of digits to any natural number starting number will have one of these outcomes:

1. Black hole 1 in one or more applications of the process.

2. The starting number is the first number of a cyclic black hole of length 8. Possible cyclic black hole starting numbers from least to greatest are: 4, 16, 20, 37, 42, 58, 89, and 145.

3. A cyclic black hole number is reached in one or more applications of the process, the cyclic black hole takes over, and the starting number does not appear again.

If $n$ is a whole number, then $10^n$ will be absorbed into black hole 1 in one application of the sum of squares of digits process. Try it with $10^0 = 1$, $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$, and so on.

If the only non-zero digits of a natural number are 1 and 3, then that number is sucked into black hole 1 in two applications of the sum of squares of digits process. Try it with 103, 130, 301, 310, 1003, 1030, 1300, et cetera, et cetera.

If the only non-zero digits of a natural number are 6 and 8, then that number is slurped into black hole 1 in two applications of the sum of squares of digits process. Try it with 68, 608, 680, 806, 860, 6008, and so on.

If the only non-zero digits of a natural number are 1 and 5, then that number is tumbles into black hole 1 in three applications of the sum of squares of digits process. Try it with 15, 105, 150, 501, 510, 1005, 5001, up, up, and away.

Numbers that begin a cyclic black hole are, in order from least to greatest:

$$4, 16, 20, 37, 42, 58, 89, \text{ and } 145$$

The number 2 reaches cyclic black hole number 4 in one application of the sum of squares of digits process. Are there other numbers that reach a cyclic black hole number in one application of the sum of squares of digits process? If yes, find one or more of these numbers.

The number 38 reaches cyclic black hole number 58 in two applications of the sum of squares of digits process ($38 \rightarrow 73 \rightarrow 58$). Are there other numbers that reach a cyclic black hole number in one application of the sum of squares of digits process? If yes, find one or more of these numbers.

Find numbers that reach a cyclic black hole number in four applications of the sum of squares of digits process. Or five applications. Or more applications.

Investigate!
The Sum of Cubes of Digits Process

The section of this book called Mathemagical Black Hole 153 explores 3-digit numbers divisible by 3 that fall into black hole 153 using the following process:

1. Start with any 3-digit number that is divisible by 3.
2. Calculate the sum of the cubes of the digits of the starting number.
3. If the number calculated in Step 2 is not equal to 153, use it again in Step 3 and continue. If the number calculated in Step is equal to 153, stop. Black hole 153.

In this section, we will apply the sum of cubes of digits process to numbers that are divisible by 3 and also to numbers that are not divisible by 3.

Example 1. The numbers 1, 10, and 100 are not divisible by 3. Each of these numbers falls into black hole 1 in one application of the sum of cubes of digits process.

<table>
<thead>
<tr>
<th>Start with 1</th>
<th>number</th>
<th>sum of cubes of digits</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

1 is a black hole using the sum of cubes of digits process.

<table>
<thead>
<tr>
<th>Start with 10</th>
<th>number</th>
<th>sum of cubes of digits</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>1 + 0</td>
<td>1</td>
</tr>
</tbody>
</table>

10 → 1 in one application of the sum of cubes of digits process.

<table>
<thead>
<tr>
<th>Start with 100</th>
<th>number</th>
<th>sum of cubes of digits</th>
<th>new number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>1 + 0 + 0</td>
<td>1</td>
</tr>
</tbody>
</table>

100 → 1 in one application of the sum of cubes of digits process.
Your Turn. Show that 1000 and 10,000 fall into black hole 1 in one application of the sum of cubes of digits process.

<table>
<thead>
<tr>
<th>Start with 1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>1000</td>
</tr>
</tbody>
</table>

1000 $\rightarrow$ 1 in one application of the sum of cubes of digits process?

<table>
<thead>
<tr>
<th>Start with 10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>10,000</td>
</tr>
</tbody>
</table>

10,000 $\rightarrow$ 1 in one application of the sum of cubes of digits process?

Answers

<table>
<thead>
<tr>
<th>Start with 1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>1000</td>
</tr>
</tbody>
</table>

1000 $\rightarrow$ 1 in one application of the sum of cubes of digits process.

<table>
<thead>
<tr>
<th>Start with 10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>10,000</td>
</tr>
</tbody>
</table>

10,000 $\rightarrow$ 1 in one application of the sum of cubes of digits process.

Conjecture. If $n$ is a whole number, then $10^n$ will fall into black hole 1 in one application of the sum of cubes of digits process. The previous examples and Your Turn exercises show that the conjecture is true for $10^0 = 1$, $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$, and $10^4 = 10,000$. What do you think? Is the conjecture true for $n$ equal to any whole number?
We know that the 3-digit number 153 is equal to the sum of the cubes of its digits:

\[ 1^3 + 5^3 + 3^3 = 1 + 125 + 27 = 153 \]  

[See Mathemagical Black Hole 153 in this book.]

We wonder – are other 3-digit numbers equal to the sums of the cubes of their digits. An Internet search produced three such numbers: 370, 371, and 407. These numbers are not divisible by 3, so they do not fall into black hole 153.

**Example 2.** The number 370 is not divisible by 3, and does not fall into black hole 153. Apply the sum of cubes of digits process to 370 and – presto! – black hole 370.

<table>
<thead>
<tr>
<th>Cubes of the decimal digits:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0^3 = 0), (1^3 = 1), (2^3 = 8), (3^3 = 27), (4^3 = 64), (5^3 = 125)</td>
</tr>
<tr>
<td>(6^3 = 216), (7^3 = 343), (8^3 = 512), (9^3 = 729)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Start with 370</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>number</strong></td>
</tr>
<tr>
<td>370</td>
</tr>
</tbody>
</table>

370 is a black hole using the sum of cubes of digits process.

**Example 3.** The number 371 is not divisible by 3, and does not fall into black hole 153. Apply the sum of cubes of digits process to 371 and – abracadabra! – black hole 371.

<table>
<thead>
<tr>
<th>Cubes of the decimal digits:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0^3 = 0), (1^3 = 1), (2^3 = 8), (3^3 = 27), (4^3 = 64), (5^3 = 125)</td>
</tr>
<tr>
<td>(6^3 = 216), (7^3 = 343), (8^3 = 512), (9^3 = 729)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Start with 371</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>number</strong></td>
</tr>
<tr>
<td>371</td>
</tr>
</tbody>
</table>

371 is a black hole using the sum of cubes of digits process.
**Your Turn.** The number 407 is not divisible by 3, and does not fall into black hole 153. Apply the sum of cubes of digits process to 407 and show that – alakazam! – black hole 407.

<table>
<thead>
<tr>
<th>Cubes of the decimal digits:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^3 = 0$, $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125$</td>
</tr>
<tr>
<td>$6^3 = 216$, $7^3 = 343$, $8^3 = 512$, $9^3 = 729$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Start with 407</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>407</td>
</tr>
</tbody>
</table>

Is 407 a black hole using the sum of cubes of digits process?

<table>
<thead>
<tr>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start with 407</td>
</tr>
<tr>
<td>number</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>407</td>
</tr>
</tbody>
</table>

407 is a black hole using the sum of cubes of digits process.

Using the sum of cubes of digits process, numbers that are divisible by 3 fall into black hole 153. Do numbers that are not divisible by 3 fall into black hole 370, or black hole 371, or black hole 407? Or might they fall into a cyclic black hole.

**Wow! Lots of questions. It is time to investigate.**

With notebook, pencil, eraser, and TI-84 in hand, we went to one of our favorite places and calculated sums of cubes of digits of numbers. We began with 1 and continued systematically with 2, 3, 4, 5, 6, et cetera, et cetera. In the handy table on the next page, we share some of our findings. To cram much information in a modicum of space, we show the sequence of numbers resulting from repeated apps (applications) of the sum of cubes of digits process. For example, here are the sequences of numbers for starting numbers 2, 3, and 4:

2 $\rightarrow$ 8 $\rightarrow$ 512 $\rightarrow$ 134 $\rightarrow$ 92 $\rightarrow$ 737 $\rightarrow$ 713 $\rightarrow$ 371 $\rightarrow$ Black hole 371 in 7 apps
3 $\rightarrow$ 27 $\rightarrow$ 351 $\rightarrow$ 153 $\rightarrow$ Black hole 153 in 3 apps
4 $\rightarrow$ 64 $\rightarrow$ 280 $\rightarrow$ 520 $\rightarrow$ 133 $\rightarrow$ 55 $\rightarrow$ 250 $\rightarrow$ 133 $\rightarrow$ Cyclic black hole
Apply the sum of cubes of digits process to numbers 1 through 20.

1 → 1
2 → 8 → 512 → 134 → 92 → 737 → 713 → 371
3 → 27 → 351 → 153
4 → 64 → 280 → 520 → 133 → 55 → 250 → 133
5 → 125 → 134 → 92 → 737 → 713 → 371
6 → 216 → 225 → 141 → 66 → 432 → 99 → 1458 → 702 → 351 → 153
7 → 343 → 118 → 514 → 190 → 730 → 370
8 → 512 → 134 → 92 → 737 → 713 → 371
9 → 729 → 1080 → 513 → 153
10 → 1
11 → 2 → 8 → 512 → 134 → 92 → 737 → 713 → 371
12 → 9 → 729 → 1080 → 513 → 153
13 → 28 → 520 → 133 → 55 → 250 → 133
14 → 65 → 341 → 92 → 737 → 713 → 371
15 → 126 → 225 → 141 → 66 → 432 → 99 → 1458 → 702 → 351 → 153
16 → 217 → 352 → 160 → 217
17 → 344 → 155 → 251 → 134 → 92 → 737 → 713 → 371
18 → 513 → 153
19 → 730 → 370
20 → 8 → 512 → 134 → 92 → 737 → 713 → 371

We hope you noticed that applying the sum of cubes of digits repeatedly to natural numbers from 1 to 20 slurped each and every number into a black hole or a cyclic black hole.

- Numbers that are divisible by 3 collect in black hole 153.
- Numbers that are not divisible by 3 end in black hole 370, black hole 371, cyclic black hole 133, 55, 250, or cyclic black hole 217, 352, 160.
- Hmmmm . . . so far no number has tumbled into black hole 407.

On the next page, we continue our investigation.
Apply the sum of cubes of digits process to numbers 21 through 40.

21 → 9 → 729 → 1080 → 513 → 153  
   Black hole 153 in 5 apps
22 → 16 → 217 → 352 → 160 → 217  
   Cyclic black hole in 2 apps
23 → 35 → 152 → 134 → 92 → 737 → 713 → 371  
   Black hole 371 in 7 apps
24 → 72 → 351 → 371  
   Black hole 153 in 3 apps
25 → 133 → 55 → 250 → 133  
   Cyclic black hole in 1 app
26 → 224 → 80 → 512 → 134 → 92 → 737 → 713 → 371  
   Black hole 371 in 8 apps
27 → 351 → 153  
   Black hole 153 in 3 apps
28 → 520 → 133 → 55 → 250 → 133  
   Cyclic black hole in 2 apps
29 → 737 → 713 → 371  
   Black hole 371 in 3 apps
30 → 27 → 351 → 153  
   Black hole 153 in 3 apps
31 → 28 → 520 → 133 → 55 → 250 → 133  
   Cyclic black hole in 3 apps
32 → 35 → 152 → 134 → 92 → 737 → 713 → 371  
   Black hole 371 in 7 apps
33 → 54 → 189 → 1242 → 81 → 513 → 153  
   Black hole 153 in 6 apps
34 → 91 → 730 → 370  
   Black hole 370 in 3 apps
35 → 152 → 134 → 92 → 737 → 713 → 371  
   Black hole 371 in 6 apps
36 → 243 → 99 → 1458 → 702 → 351 → 153  
   Black hole 153 in 6 apps
37 → 370  
   Black hole 370 in 1 app
38 → 539 → 881 → 1025 → 134 → 92 → 737 → 713 → 371  
   Black hole 371 in 8 apps
39 → 756 → 684 → 792 → 1080 → 513 → 153  
   Black hole 153 in 6 apps
40 → 64 → 280 → 520 → 133 → 55 → 250 → 133  
   Cyclic black hole in 4 apps

Applying the sum of cubes of digits repeatedly to natural numbers from 21 to 40 guzzled each and every number into a black hole or a cyclic black hole.

- Numbers that are divisible by 3 collect in black hole 153.
- Numbers that are not divisible by 3 are more diverse in their ultimate fate. They end in black hole 370, black hole 371, or cyclic black hole 133, 55, 250, or cyclic black hole 217, 352, 160.
- So far no number has tumbled into black hole 407.

On the next page, we continue our investigation.
Apply the sum of cubes of digits process to numbers 41 to 49.

41 → 65 → 341 → 92 → 737 → 713 → 371
42 → 72 → 351 → **153**
43 → 91 → 730 → **370**
44 → 128 → 521 → 134 → 92 → 737 → 713 → **371**
45 → 189 → 1242 → 81 → 513 → **153**
46 → 280 → 520 → **133** → 55 → 250 → **133**
47 → **407**
48 → 576 → 684 → 792 → 1080 → 513 → **153**
49 → 793 → 1099 → **1459** → 919 → **1459**

Applying the sum of cubes of digits repeatedly to natural numbers from 41 to 49 pushed each and every number into a black hole or a cyclic black hole.

- Numbers that are divisible by 3 collect in black hole 153.
- Numbers that are not divisible by 3 are more diverse in their ultimate fate. They end in black hole 370, black hole 371, or a cyclic black hole.
- We have encountered cyclic black hole 133, 55, 250, cyclic black hole 217, 352, 160, and cyclic black hole 1459, 919.
- The number 47 tumbled into black hole 407.

Many numbers from 50 on are repeats of numbers from 1 to 49. The number 50 has the same sequence as the number 5. Permutations have the same digits, and so have the same sum of cubes of digits. 51 is a repeat of 15, 52 is a repeat of 25, 53 is a repeat of 35, and 54 is a repeat of 45.

We continue our investigation on the next page with numbers that are not repeats of numbers from 1 to 49.
Apply the sum of cubes of digits process to selected numbers from 55 to 99

55 → 250 → 133 → **55**  
Cyclic black hole
66 → 341 → 92 → 737 → 713 → **371**  
Black hole 371 in 5 apps
57 → 468 → 792 → 1080 → 513 → **153**  
Black hole 153 in 5 apps
58 → 637 → 586 → 853 → 664 → 496 → 1009 → 730 → **370**  
Black hole 370 in 8 apps
59 → 854 → 701 → 344 → 155 → 251 → 134 → 92 → 737  
Black hole 371 in 10 apps
67 → 559 → 979 → 1801 → 514 → 190 → 730 → **370**  
Black hole 370 in 7 apps
68 → 728 → 863 → 755 → 593 → 881 → 1025 → 134 → 92  
Black hole 371 in 11 apps
69 → 945 → 918 → 1242 → 81 → 513 → **153**  
Black hole 153 in 6 apps
77 → 686 → 944 → 857 → 980 → 1241 → 74 → **407**  
Black hole 407 in 7 apps
78 → 855 → 762 → 567 → 684 → 792 → 1080 → 513 → **153**  
Black hole 153 in 8 apps
79 → 1072 → **352** → 160 → 217 → **352**  
Cyclic black hole in 2 apps
88 → 1024 → 73 → **370**  
Black hole 370 in 3 apps
89 → 1241 → 74 → **407**  
Black hole 407 in 3 apps
99 → 1458 → 702 → 351 → **153**  
Black hole 153 in 4 apps

Whew! In the above tables you can find what happens to numbers from 1 to 99 inclusive as we repeatedly apply the sum of cubes of digits process. We omitted numbers that are reverses of numbers already in the table. Numbers that are reverses of each other have the same sums of cubes of digits and therefore the same sums of cubes of digits.

- The number 1 is a black hole.
- Numbers divisible by 3 end up in black hole 153.
- Numbers that are not divisible by 3 make their ways to one of these black holes:

  Black hole 370  
  Black hole 371  
  Black hole 407  
  Cyclic black hole of length 3: 133, 55, 250  
  Cyclic black hole of length 3: 55, 250, 133  
  Cyclic black hole of length 3: 217, 352, 160  
  Cyclic black hole of length 3: 352, 160, 217  
  Cyclic black hole of length 2: 1459, 919.
Your Turn: Investigate

We investigated numbers from 1 to 99. We encourage you to continue with 100, 101, 102, 103, and so on. Here are some ideas that might help your investigations.

101 → $1^3 + 0^3 + 1^3 = 2$. The rest of the sequence is in the 2 row of the 1 to 40 table.
102 → $1^3 + 0^3 + 2^3 = 9$. The rest of the sequence is in the 9 row of the 1 to 40 table.
111 → $1^3 + 1^3 + 1^3 = 3$. The rest of the sequence is in the 3 row of the 1 to 40 table.
112 → 10 → 1. Black hole. Permutations 121, 212, and 221 also go to black hole 1.
136 → 244 and 244 → 136. This a cyclic black hole pair.
1459 → 919 and 919 → 1459. This is a cyclic black hole pair

Are there other cyclic black hole pairs?

133, 55, and 250 are a cyclic black hole triple. Start with one of these numbers and you will get the other two.

217, 352, and 160 are a cyclic black hole triple. Start with one of these numbers and you will get the other two.

Are there other cyclic black hole triples?

Classroom Project

Sort the numbers from 100 to 999 into categories such as these:

- Falls into black hole 1.
- Falls into black hole 153.
- Falls into black hole 370.
- Falls into black hole 371.
- Falls into black hole 407.
- Falls into a cyclic black hole.

Form teams of students and assign each team part of the task.

Investigate!

We have reached the end of this Mathemagical Black Holes adventure. We hope that you found some joy in learning about mathemagical black holes.